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Calculation of C5 Stresses and Deflections in C5 Due to Thermal and Pressure Loading

High Energy Physics Division

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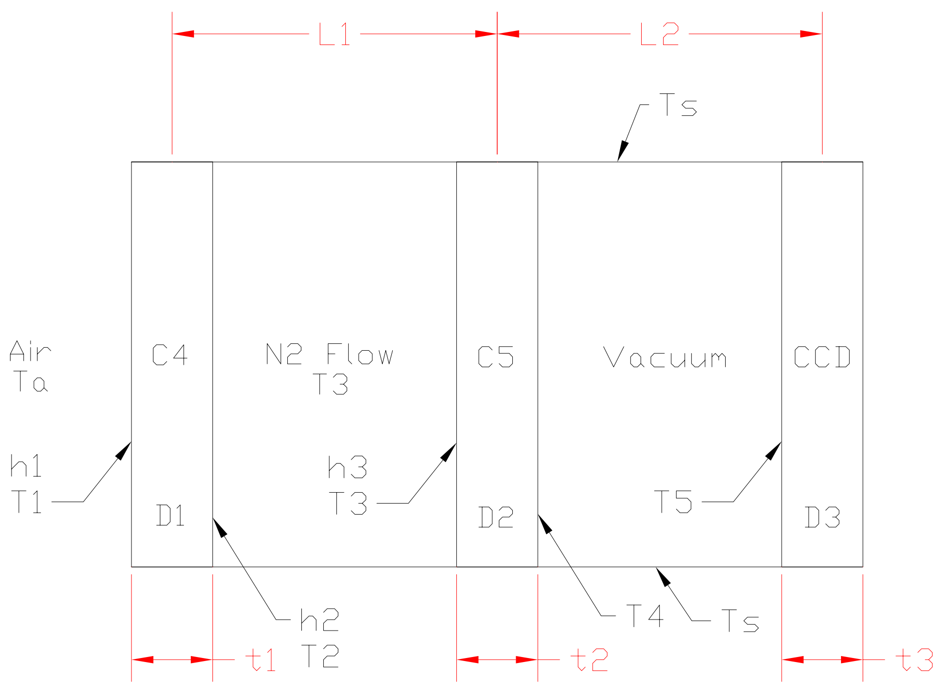
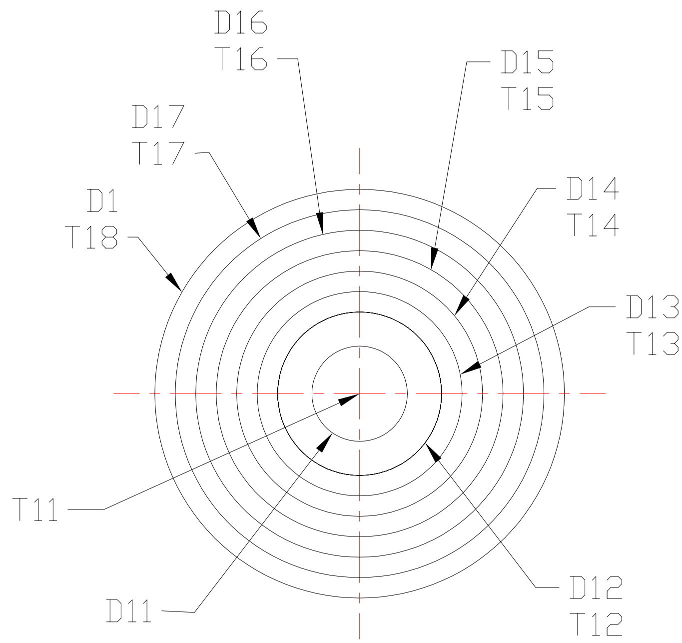
Calculation of C5 Stresses and Deflections Due to Applied Pressure and Temperature Difference

Sections:
1.0 Calculation of the Lens Temperature Distributions
2.0 Calculation of the Lens Stresses and Deflections Due to Temp. Differences
3.0 Calculation of the Lens Stresses and Deflections Due to 1atm. Pressure
4.0 Calculation of Combined Lens Stresses and Deflections Due to Temperature and Pressure
5.0 Comparison with FEA model and Conclusions

References:
Roark's Formulas for Stress and Strain -- Table 24
Advanced Mechanics of Materials -- Chapter 4
Fundamentals of Heat Transfer, DeWitt

1.0 Calculation of Lens Temperature Differences





Schematic showing the Diameter and Temp Locations in Thermal Model For Surface T1 of C4

As an initial attempt to understand the parameters influencing the thermal equilibrium of the camera/lens assembly a simple finite difference model has been created. This model examines the heat flux through the lenses and the region of N2 flow. The following assumptions have been made:

1. All lenses are modeled as simple disks.
2. Each disk is subdivided into eight concentric rings and split in half.
3. That heat can flow radially through contact with the outer cylinder. That contact between the lens and outer cylinder is through two o-rings that is t4 for C4 and t5 for C5 thick.
4. The heat flow into each segment is calculated and the individual temperatures then solved for.
5. It is assumed that there is conduction between each lens and the outer cylinder; that there is radiation heat transfer between the CCD and the T4 surface of C5; that there is free and forced convection between the N2 gas flow and the T3 surface of C5 and the T2 surface of C4; that there is free convection between the atmospheric air and the T1 surface of C4. Also considered is radiation between the T2 surface of C4 and the T3 surface of C5 as well as radiation between the outer radial surface of C4 and C5 and the support structure.

The inputs into the system are:

1. N2 flow rate
2. N2 inlet temperature taken as the atmospheric temperature.
3. Ta, the atmospheric temperature.
4. Ts, the temperature of the outer cylinder.
5. T5, the temperature of the CCD surface.

The outputs of the calculation are:

1. The temperature distribution of the lense surfaces, T1, T2, T3, T4 (see schematic above)
2. The average temperature of the N2. Tn is the average of the inlet and outlet temps and is used in the calculations. The defined inlet temperature and Tn are then used to calculate the outlet temp, To

1.1 Define Inputs

1.1.1 Geometric Inputs

$D1 := 604mm$	Diameter of C4 Lense
$D2 := 542mm$	Diameter of C5 Lense
$D3 := 508mm$	Diameter of CCD
$t1 := 49.83mm$	Thickness of C4 Lense
$t2 := 55.1mm$	Thickness of C5 Lense
$t3 := 20mm$	Thickness of CCD
$t4 := 9.5mm$	Thickness of intertace between C4 and outer cylinder
$t5 := 9.5mm$	Thickness of interface between C5 and Cylinder
$L1 := 144.77mm$	Distance between C4-C5
$L2 := 30mm$	Distance between C5-CCD

1.1.2 Temperature Inputs

$Ta := 25K + 273K$	Ambient Air Temp
$Ts := 25K + 273K$	Outer steel Temp
$T5 := 173K$	CCD surface Temp.
$Ti := 25K + 273K$	Inlet Temp of N2

1.1.3 Properties of Air

$kJ := 1000J$

$\mu a := 184.6 \times 10^{-7} \dfrac{kg}{s \cdot m}$	Viscosity of air
$cpa := 1.007 \dfrac{kJ}{kg \cdot K}$	Specific Heat at constant pressure
$ka := 0.0263 \dfrac{W}{m \cdot K}$	Thermal Conductivity
$\nu a := 15.87 \times 10^{-6} \dfrac{m^2}{s}$	Kinematic viscosity

1.1.4 Properties of N2

$\mu N2 := 178.2 \times 10^{-7} \frac{N \cdot s}{m^2}$ Viscosity of N2

$cpN2 := 1.041 \frac{kJ}{kg \cdot K}$ Specific Heat at constant pressure

$kN2 := 25.9 \times 10^{-3} \frac{W}{m \cdot K}$ Thermal Conductivity

$\nu N2 := 15.86 \times 10^{-6} \cdot \frac{m^2}{s}$ Kinematic viscosity

$v := 3.0 \frac{ft^3}{min}$ Volume Flow Rate of N2

$\rho N2 := 1.1233 \frac{kg}{m^3}$ Density of N2 at 27C

$mN2 := \rho N2 \cdot v$ $mN2 = 0.002 \frac{kg}{s}$ Mass flow rate of N2

$V := \frac{v}{\frac{(D1 + D2)}{2} \cdot L1}$ $V = 0.017 \frac{m}{s}$ Velocity of N2 over C4 and C5

1.1.5 Thermal Properties of Lense and CCD

$k1 := 0.13 \frac{W}{m \cdot K}$ Thermal conductivity of Silicone ring at outer perimeter of C4

$k2 := 0.13 \frac{W}{m \cdot K}$ Thermal conductivity of Silicone ring at outer perimeter of C5

$k3 := 1.05 \frac{W}{m \cdot K}$ Thermal conductivity of glass in C4

$k4 := 1.05 \frac{W}{m \cdot K}$ Thermal Conductivity of Glass in C5

$\varepsilon 1 := 0.85$ Emissivity of C5 surface

$\varepsilon 2 := 0.85$ Emissivity of CCD surface

$\sigma := 5.669 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$ Stefan Boltzman Constant

$\frac{D2}{L2} = 18.067$

$F12 := 0.28$ Radiation Shape Factor based on the ratio of the C5 diameter to the separation distance

1.2.0 Calculation of Convection Coefficients

1.2.1 Calculation of free convection coefficient, h1, on C4

This calculation is based on an assumed temperature for the outer surface of C4. After the final calculation of the temperature distribution, go back and check if this assumption was correct.

$Pr := \frac{\mu a \cdot cpa}{ka}$ $Pr = 0.707$ Prandlt Number

$T1 := 15K + 273K$ Assumed temperature of outer surface of C4

$T1 = 288.000\,K$

$\beta := \frac{1}{T1}$ $\beta = 0.00347\,\frac{1}{K}$ Volumetric Coefficient of thermal expansion for an ideal gas

$g := 9.81\,\frac{m}{s^2}$ Gravity acceleration

$L := \sqrt{\frac{D1^2}{2}}$ $L = 0.43\,m$ Effective Length

$Gr := \left|\frac{g\cdot\beta\cdot L^3\cdot(T1 - Ta)}{2}\right|$ $Gr = 105363087.505$ Grashof Number

$$h1 := \frac{ka}{L}\cdot 0.678\cdot\left(\frac{Pr^2}{0.952 + Pr}\right)^{\frac{1}{4}}\cdot Gr^{\frac{1}{4}}$$

$h1 = 3.134\,\frac{W}{m^2\cdot K}$ Free Convection Coefficient

1.2.2 Calculation of forced convection coeficient between C4-C5 ---- h2

$Pr := \frac{\mu N2\cdot cpN2}{kN2}$ $Pr = 0.716$ Prandlt Number

$L := \sqrt{\frac{D1^2}{2}}$ $L = 0.43\,m$ Effective Length

$Re := \frac{V\cdot L}{\nu N2}$ $Re = 459.622$ Reynolds Number

$$h2a := \frac{kN2}{L} \cdot 0.664 \cdot Pr^{\frac{1}{3}} \cdot Re^{\frac{1}{2}}$$

$$h2a = 0.772 \frac{W}{m^2 \cdot K}$$ Convection Coefficient on surface of C5 at N2 interface

$$Pr := \frac{\mu a \cdot cpN2}{kN2}$$ $Pr = 0.742$ Prandlt Number

$$T2 := 10K + 273K$$ Assumed temperature of inner surface of C4

$$T2 = 283.000\,K$$

$$\beta := \frac{1}{T2}$$ $\beta = 0.00353 \frac{1}{K}$ Volumetric Coefficient of thermal expansion for an ideal gas

$$g := 9.81 \frac{m}{s^2}$$ Gravity acceleration

$$L := \sqrt{\frac{D1^2}{2}}$$ $L = 0.43\,m$ Effective Length

$$Gr := \left| \frac{g \cdot \beta \cdot L^3 \cdot (T2 - Ti)}{2} \right|$$ $Gr = 160836939.230$ Grashof Number

$$h2b := \frac{ka}{L} \cdot 0.678 \cdot \left(\frac{Pr^2}{0.952 + Pr} \right)^{\frac{1}{4}} \cdot Gr^{\frac{1}{4}}$$

$$h2b = 3.550 \frac{W}{m^2 \cdot K}$$ Free Convection Coefficient

$$h2 := h2a + h2b$$ $h2 = 4.322 \frac{W}{m^2 \cdot K}$ Total Convection Coefficient

1.2.3 Calculation of forced convection coeficient between C4-C5 ---- h3

$$Pr := \frac{\mu N2 \cdot cpN2}{kN2}$$ $Pr = 0.716$ Prandlt Number

$$L := \sqrt{\frac{D2^2}{2}}$$ $L = 0.38\,m$ Effective Length

$Re := \frac{V \cdot L}{\nu N2}$ $\quad Re = 412.442$ Reynolds Number

$$h3a := \frac{kN2}{L} \cdot 0.664 \cdot Pr^{\frac{1}{3}} \cdot Re^{\frac{1}{2}}$$

$h3a = 0.815 \frac{W}{m^2 \cdot K}$ Convection Coefficient on surface of C5 at N2 interface

$Pr := \frac{\mu a \cdot cpN2}{kN2}$ $\quad Pr = 0.742$ Prandlt Number

$T3 := -20K + 273K$ Assumed temperature of outer surface of C5

$T3 = 253.000\,K$

$\beta := \frac{1}{T3}$ $\quad \beta = 0.00395 \frac{1}{K}$ Volumetric Coefficient of thermal expansion for an ideal gas

$g := 9.81 \frac{m}{s^2}$ Gravity acceleration

$L := \sqrt{\frac{D2^2}{2}}$ $\quad L = 0.38\,m$ Effective Length

$Gr := \left| \frac{g \cdot \beta \cdot L^3 \cdot (T3 - Ti)}{2} \right|$ $\quad Gr = 389995874.179$ Grashof Number

$$h3b := \frac{ka}{L} \cdot 0.678 \cdot \left(\frac{Pr^2}{0.952 + Pr} \right)^{\frac{1}{4}} \cdot Gr^{\frac{1}{4}}$$

$h3b = 4.937 \frac{W}{m^2 \cdot K}$ Free Convection Coefficient

$h3 := h3a + h3b$ $\quad h3 = 5.752 \frac{W}{m^2 \cdot K}$ Total Convection Coefficient

1.3.0 Calculation of Finite Difference Geometry

Define Diameters of Sections

$D1c4 := .1 \cdot D1$ $\quad D1c4 = 60.400\,mm$ Diameter of inner section

$D2c4 := 0.3 \cdot D1$ $\quad D2c4 = 181.200\,mm$

$D3c4 := 0.5D1$ $D3c4 = 302.000\,mm$

$D4c4 := 0.60 \cdot D1$ $D4c4 = 362.400\,mm$

$D5c4 := 0.70 \cdot D1$ $D5c4 = 422.800\,mm$

$D6c4 := 0.80 \cdot D1$ $D6c4 = 483.200\,mm$

$D7c4 := 0.90 \cdot D1$ $D7c4 = 543.600\,mm$

$D8c4 := 1.0 \cdot D1$ $D8c4 = 604.000\,mm$

$D1c5 := 0.1D2$ $D1c5 = 54.200\,mm$

$D2c5 := 0.3 \cdot D2$ $D2c5 = 162.600\,mm$

$D3c5 := 0.5 \cdot D2$ $D3c5 = 271.000\,mm$

$D4c5 := 0.60 \cdot D2$ $D4c5 = 325.200\,mm$

$D5c5 := 0.70 \cdot D2$ $D5c5 = 0.379\,m$

$D6c5 := 0.8D2$ $D6c5 = 0.434\,m$

$D7c5 := 0.90 \cdot D2$ $D7c5 = 0.488\,m$

$D8c5 := 1.0 \cdot D2$ $D8c5 = 0.542\,m$

Calculate Areas of Conduction

$A1c4 := \pi \cdot \frac{D1c4^2}{4}$ $A1c4 = 0.003\,m^2$ Surface area of Section 1 normal to the axis

$$A2c4 := \pi \cdot \frac{\left[\frac{(D2c4 + D3c4)}{2}\right]^2 - D1c4^2}{4}$$ $A2c4 = 0.043\,m^2$

$$A3c4 := \pi \cdot \frac{\left[\frac{(D4c4 + D3c4)}{2}\right]^2 - \left[\frac{(D3c4 + D2c4)}{2}\right]^2}{4}$$ $A3c4 = 0.041\,m^2$

$$A4c4 := \pi \cdot \frac{\left[\frac{(D5c4 + D4c4)}{2}\right]^2 - \left[\frac{(D4c4 + D3c4)}{2}\right]^2}{4}$$ $A4c4 = 0.034\,m^2$

$$A5c4 := \pi \cdot \frac{\left[\frac{(D6c4 + D5c4)}{2}\right]^2 - \left[\frac{(D5c4 + D4c4)}{2}\right]^2}{4}$$ $A5c4 = 0.040\,m^2$

$$A6c4 := \pi \cdot \frac{\left[\frac{(D7c4 + D6c4)}{2}\right]^2 - \left[\frac{(D6c4 + D5c4)}{2}\right]^2}{4}$$ $A6c4 = 0.046\,m^2$

$$A7c4 := \pi \cdot \frac{\left[\frac{(D8c4 + D7c4)}{2}\right]^2 - \left[\frac{(D7c4 + D6c4)}{2}\right]^2}{4}$$ $A7c4 = 0.052\,m^2$

$$A8c4 := \pi \cdot \frac{D8c4^2 - \left[\frac{(D8c4 + D7c4)}{2}\right]^2}{4}$$ $A8c4 = 0.028\,m^2$

$$A1c5 := \pi \cdot \frac{D1c5^2}{4}$$ $A1c5 = 0.002\,m^2$ Surface area of Section 1 normal to the axis

$$A2c5 := \pi \cdot \frac{\left[\frac{(D2c5 + D3c5)}{2}\right]^2 - D1c5^2}{4}$$ $A2c4 = 0.043\,m^2$

$$A3c5 := \pi \cdot \frac{\left[\frac{(D4c5 + D3c5)}{2}\right]^2 - \left[\frac{(D3c5 + D2c5)}{2}\right]^2}{4}$$ $A3c4 = 0.041\,m^2$

$$A4c5 := \pi \cdot \frac{\left[\frac{(D5c5 + D4c5)}{2}\right]^2 - \left[\frac{(D4c5 + D3c5)}{2}\right]^2}{4}$$ $A4c4 = 0.034\,m^2$

$$A5c5 := \pi \cdot \frac{\left[\frac{(D6c5 + D5c5)}{2}\right]^2 - \left[\frac{(D5c5 + D4c5)}{2}\right]^2}{4}$$ $A5c4 = 0.040\,m^2$

$$A6c5 := \pi \cdot \frac{\left[\frac{(D7c5 + D6c5)}{2}\right]^2 - \left[\frac{(D6c5 + D5c5)}{2}\right]^2}{4}$$ $A6c4 = 0.046\,m^2$

$$A7c5 := \pi \cdot \frac{\left[\frac{(D8c5 + D7c5)}{2}\right]^2 - \left[\frac{(D7c5 + D6c5)}{2}\right]^2}{4}$$ $A7c4 = 0.052\,m^2$

$$A8c5 := \pi \cdot \frac{D8c5^2 - \left[\frac{(D8c5 + D7c5)}{2}\right]^2}{4} \qquad A8c4 = 0.028\,m^2$$

Calculation of Areas in Contact between Rings

$$As1c4 := \frac{\pi \cdot D1c4 \cdot t1}{2} \qquad As1c4 = 0.005\,m^2$$

Area of outer perimeter between layers

$$As2c4 := \frac{\pi \cdot \frac{(D3c4 + D2c4)}{2} \cdot t1}{2} \qquad As2c4 = 0.019\,m^2$$

$$As3c4 := \frac{\pi \cdot \frac{(D4c4 + D3c4)}{2} \cdot t1}{2} \qquad As3c4 = 0.026\,m^2$$

$$As4c4 := \frac{\pi \cdot \frac{(D5c4 + D4c4)}{2} \cdot t1}{2} \qquad As4c4 = 0.031\,m^2$$

$$As5c4 := \frac{\pi \cdot \frac{(D6c4 + D5c4)}{2} \cdot t1}{2} \qquad As5c4 = 0.035\,m^2$$

$$As6c4 := \frac{\pi \cdot \frac{(D7c4 + D6c4)}{2} \cdot t1}{2} \qquad As6c4 = 0.040\,m^2$$

$$As7c4 := \frac{\pi \cdot \frac{(D8c4 + D7c4)}{2} \cdot t1}{2} \qquad As7c4 = 0.045\,m^2$$

$$As8c4 := \frac{\pi \cdot D8c4 \cdot t4}{2} \qquad As8c4 = 0.009\,m^2$$

Area of outer perimeter surface of C4 in contact with outer cylinder through two o-rings

$$As1c5 := \frac{\pi D1c5 \cdot t2}{2} \qquad As1c5 = 0.005\,m^2$$

$$As2c5 := \frac{\pi \cdot \frac{(D3c5 + D2c5)}{2} \cdot t2}{2} \qquad As2c5 = 0.019\,m^2$$

$$As3c5 := \frac{\pi \cdot \frac{(D4c5 + D3c5)}{2} \cdot t2}{2} \qquad As3c5 = 0.026\,m^2$$

$$As4c5 := \frac{\pi \cdot \frac{(D5c5 + D4c5)}{2} \cdot t2}{2} \qquad As4c5 = 0.030\,m^2$$

$$As5c5 := \frac{\pi \cdot \frac{(D6c5 + D5c5)}{2} \cdot t2}{2} \qquad As5c5 = 0.035\,m^2$$

$$As6c5 := \frac{\pi \cdot \frac{(D7c5 + D6c5)}{2} \cdot t2}{2} \qquad As6c5 = 0.040\,m^2$$

$$As7c5 := \frac{\pi \cdot \frac{(D8c5 + D7c5)}{2} \cdot t2}{2} \qquad As7c5 = 0.045\,m^2$$

$$As8c5 := \frac{\pi \cdot D8c5 \cdot t5}{2} \qquad As8c5 = 0.008\,m^2$$

Area of outer perimeter surface of C5 in contact wtih outer cylinder through two O-rings

1.4.0 Calculation of Temperatures and Thermal Equilibrium

Define initial guesses for Temperatures.

$T1_0 := 285K$ $\quad T1_1 := 285K$ $\quad T1_2 := 285K$ $\quad T1_3 := 285K$

$T1_4 := 285K$ $\quad T1_5 := 285K$ $\quad T1_6 := 285K$ $\quad T1_7 := 285K$

$T2_0 := 280K$ $\quad T2_1 := 280K$ $\quad T2_2 := 280K$ $\quad T2_3 := 280K$

$T2_4 := 280K$ $\quad T2_5 := 280K$ $\quad T2_6 := 280K$ $\quad T2_7 := 280K$

$T3_0 := 245K$ $\quad T3_1 := 245K$ $\quad T3_2 := 245K$ $\quad T3_3 := 245K$

$T3_4 := 245K$ $\quad T3_5 := 245K$ $\quad T3_6 := 245K$ $\quad T3_7 := 245K$

$T4_0 := 240K$ $T4_1 := 240K$ $T4_2 := 240K$ $T4_3 := 240K$

$T4_4 := 240K$ $T4_5 := 240K$ $T4_6 := 240K$ $T4_7 := 240K$

$Tn := 260K$

Given

Thermal Equilibrium on surface #1 of C4

$$h1 \cdot A1c4 \cdot (Ta - T1_0) - \frac{k3 \cdot A1c4}{t1} \cdot (T1_0 - T2_0) - \frac{k3 \cdot As1c4}{\left(\frac{D2c4}{2}\right)} \cdot (T1_0 - T1_1) = 0.0$$

$$h1 \cdot A2c4 \cdot (Ta - T1_1) - \frac{k3 \cdot A2c4}{t1} \cdot (T1_1 - T2_1) - \frac{k3 \cdot As2c4}{\left(\frac{D3c4 - D2c4}{2}\right)} \cdot (T1_1 - T1_2) + \frac{k3 \cdot As1c4}{\left(\frac{D2c4}{2}\right)} \cdot (T1_0 - T1_1) = 0.0$$

$$h1 \cdot A3c4 \cdot (Ta - T1_2) - \frac{k3 \cdot A3c4}{t1} \cdot (T1_2 - T2_2) - \frac{k3 \cdot As3c4}{\left(\frac{D4c4 - D3c4}{2}\right)} \cdot (T1_2 - T1_3) + \frac{k3 \cdot As2c4}{\left(\frac{D3c4 - D2c4}{2}\right)} \cdot (T1_1 - T1_2) = 0.0$$

$$h1 \cdot A4c4 \cdot (Ta - T1_3) - \frac{k3 \cdot A4c4}{t1} \cdot (T1_3 - T2_3) - \frac{k3 \cdot As4c4}{\left(\frac{D5c4 - D4c4}{2}\right)} \cdot (T1_3 - T1_4) + \frac{k3 \cdot As3c4}{\left(\frac{D4c4 - D3c4}{2}\right)} \cdot (T1_2 - T1_3) = 0.0$$

$$h1 \cdot A5c4 \cdot (Ta - T1_4) - \frac{k3 \cdot A5c4}{t1} \cdot (T1_4 - T2_4) - \frac{k3 \cdot As5c4}{\left(\frac{D6c4 - D5c4}{2}\right)} \cdot (T1_4 - T1_5) + \frac{k3 \cdot As4c4}{\left(\frac{D5c4 - D4c4}{2}\right)} \cdot (T1_3 - T1_4) = 0.0$$

$$h1 \cdot A6c4 \cdot (Ta - T1_5) - \frac{k3 \cdot A6c4}{t1} \cdot (T1_5 - T2_5) - \frac{k3 \cdot As6c4}{\left(\frac{D7c4 - D6c4}{2}\right)} \cdot (T1_5 - T1_6) + \frac{k3 \cdot As5c4}{\left(\frac{D6c4 - D5c4}{2}\right)} \cdot (T1_4 - T1_5) = 0.0$$

$$h1 \cdot A7c4 \cdot (Ta - T1_6) - \frac{k3 \cdot A7c4}{t1} \cdot (T1_6 - T2_6) - \frac{k3 \cdot As7c4}{\left(\frac{D8c4 - D7c4}{2}\right)} \cdot (T1_6 - T1_7) + \frac{k3 \cdot As6c4}{\left(\frac{D7c4 - D6c4}{2}\right)} \cdot (T1_5 - T1_6) = 0.0$$

$$h1 \cdot A8c4 \cdot (Ta - T1_7) - \frac{k3 \cdot A8c4}{t1} \cdot (T1_7 - T2_7) \ldots = 0.0$$
$$+ \frac{k3 \cdot As7c4}{\left(\frac{D8c4 - D7c4}{2}\right)} \cdot (T1_6 - T1_7) - \frac{k1 \cdot As8c4}{t4} \cdot (T1_7 - Ts) - \varepsilon 1 \cdot \frac{\pi \cdot D8c4 \cdot t1}{2} \cdot \sigma \cdot \left[(T1_7)^4 - (Ts)^4\right]$$

Thermal Equilibrium of Surface #2 of C4

$$\frac{k3\cdot A1c4}{t1}\left(T1_0 - T2_0\right) - h2\cdot A1c4\cdot\left(T2_0 - Tn\right) - \frac{k3\cdot As1c4}{\left(\frac{D2c4}{2}\right)}\cdot\left(T2_0 - T2_1\right) - \varepsilon 1\cdot A1c4\cdot\sigma\cdot\left[\left(T2_0\right)^4 - \left(T3_0\right)^4\right] = 0.0$$

$$\frac{k3\cdot A2c4}{t1}\cdot\left(T1_1 - T2_1\right) - h2\cdot A2c4\cdot\left(T2_1 - Tn\right) + \frac{k3\cdot As1c4}{\left(\frac{D2c4}{2}\right)}\cdot\left(T2_0 - T2_1\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As2c4}{\left(\frac{D3c4 - D2c4}{2}\right)}\cdot\left(T2_1 - T2_2\right) - \varepsilon 1\cdot A2c4\cdot\sigma\cdot\left[\left(T2_1\right)^4 - \left(T3_1\right)^4\right]$$

$$\frac{k3\cdot A3c4}{t1}\cdot\left(T1_2 - T2_2\right) - h2\cdot A3c4\cdot\left(T2_2 - Tn\right) + \frac{k3\cdot As2c4}{\left(\frac{D3c4 - D2c4}{2}\right)}\cdot\left(T2_1 - T2_2\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As3c4}{\left(\frac{D4c4 - D3c4}{2}\right)}\cdot\left(T2_2 - T2_3\right) - \varepsilon 1\cdot A3c4\cdot\sigma\cdot\left[\left(T2_2\right)^4 - \left(T3_2\right)^4\right]$$

$$\frac{k3\cdot A4c4}{t1}\cdot\left(T1_3 - T2_3\right) - h2\cdot A4c4\cdot\left(T2_3 - Tn\right) + \frac{k3\cdot As3c4}{\left(\frac{D4c4 - D3c4}{2}\right)}\cdot\left(T2_2 - T2_3\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As4c4}{\left(\frac{D5c4 - D4c4}{2}\right)}\cdot\left(T2_3 - T2_4\right) - \varepsilon 1\cdot A4c4\cdot\sigma\cdot\left[\left(T2_3\right)^4 - \left(T3_3\right)^4\right]$$

$$\frac{k3\cdot A5c4}{t1}\cdot\left(T1_4 - T2_4\right) - h2\cdot A5c4\cdot\left(T2_4 - Tn\right) + \frac{k3\cdot As4c4}{\left(\frac{D5c4 - D4c4}{2}\right)}\cdot\left(T2_3 - T2_4\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As5c4}{\left(\frac{D6c4 - D5c4}{2}\right)}\cdot\left(T2_4 - T2_5\right) - \varepsilon 1\cdot A5c4\cdot\sigma\cdot\left[\left(T2_4\right)^4 - \left(T3_4\right)^4\right]$$

$$\frac{k3\cdot A6c4}{t1}\cdot\left(T1_5 - T2_5\right) - h2\cdot A6c4\cdot\left(T2_5 - Tn\right) + \frac{k3\cdot As5c4}{\left(\frac{D6c4 - D5c4}{2}\right)}\cdot\left(T2_4 - T2_5\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As6c4}{\left(\frac{D7c4 - D6c4}{2}\right)}\cdot\left(T2_5 - T2_6\right) - \varepsilon 1\cdot A6c4\cdot\sigma\cdot\left[\left(T2_5\right)^4 - \left(T3_5\right)^4\right]$$

$$\frac{k3\cdot A7c4}{t1}\cdot\left(T1_6 - T2_6\right) - h2\cdot A7c4\cdot\left(T2_6 - Tn\right) + \frac{k3\cdot As6c4}{\left(\frac{D7c4 - D6c4}{2}\right)}\cdot\left(T2_5 - T2_6\right) \ldots = 0.0$$
$$+ \frac{-k3\cdot As7c4}{\left(\frac{D8c4 - D7c4}{2}\right)}\cdot\left(T2_6 - T2_7\right) - \varepsilon1\cdot A7c4\cdot\sigma\cdot\left[\left(T2_6\right)^4 - \left(T3_6\right)^4\right]$$

$$\frac{k3\,A8c4}{t1}\cdot\left(T1_7 - T2_7\right) - h2\cdot A8c4\cdot\left(T2_7 - Tn\right) + \frac{k3\cdot As7c4}{\left(\frac{D8c4 - D7c4}{2}\right)}\cdot\left(T2_6 - T2_7\right) \ldots = 0.0$$
$$+ \frac{-k1\cdot As8c4}{t4}\cdot\left(T2_7 - Ts\right) - \varepsilon1\cdot A8c4\cdot\sigma\cdot\left[\left(T2_7\right)^4 - \left(T3_7\right)^4\right] - \varepsilon1\cdot\frac{\pi\cdot D8c4\cdot t1}{2}\cdot\sigma\cdot\left[\left(T2_7\right)^4 - (Ts)^4\right]$$

Thermal Equilibrium of N2 Region

$$h2\cdot A1c4\cdot\left(T2_0 - Tn\right) + h2\cdot A2c4\cdot\left(T2_1 - Tn\right) + h2\cdot A3c4\cdot\left(T2_2 - Tn\right) \ldots = 0.0$$
$$+ h2\cdot A4c4\cdot\left(T2_3 - Tn\right) + h2\cdot A5c4\cdot\left(T2_4 - Tn\right) \ldots$$
$$+ h2\cdot A6c4\cdot\left(T2_5 - Tn\right) + h2\cdot A7c4\cdot\left(T2_6 - Tn\right) + h2\cdot A8c4\cdot\left(T2_7 - Tn\right) \ldots$$
$$+ h3\cdot A1c5\cdot\left(T3_0 - Tn\right) + h3\cdot A2c5\cdot\left(T3_1 - Tn\right) \ldots$$
$$+ h3\cdot A3c5\cdot\left(T3_2 - Tn\right) + h3\cdot A4c5\cdot\left(T3_3 - Tn\right) + h3\cdot A5c5\cdot\left(T3_4 - Tn\right) + h3\cdot A6c5\cdot\left(T3_5 - Tn\right) \ldots$$
$$+ h3\cdot A7c5\cdot\left(T3_6 - Tn\right) + h3\cdot A8c5\cdot\left(T3_7 - Tn\right) + mN2\cdot cpN2\cdot 2\cdot(Ti - Tn)$$

Thermal Equilibrium of Surface #3 of C5

$$h3\cdot A1c5\cdot\left(Tn - T3_0\right) - \frac{k4\cdot A1c5}{t2}\cdot\left(T3_0 - T4_0\right) - \frac{k4\cdot As1c5}{\left(\frac{D2c5}{2}\right)}\cdot\left(T3_0 - T3_1\right) + \varepsilon1\cdot A1c5\cdot\sigma\cdot\left[\left(T2_0\right)^4 - \left(T3_0\right)^4\right] = 0.0$$

$$h3\cdot A2c5\cdot\left(Tn - T3_1\right) - \frac{k4\cdot A2c5}{t2}\cdot\left(T3_1 - T4_1\right) - \frac{k4\cdot As2c5}{\left(\frac{D3c5 - D2c5}{2}\right)}\cdot\left(T3_1 - T3_2\right) \ldots = 0.0$$
$$+ \frac{k4\cdot As1c5}{\left(\frac{D2c5}{2}\right)}\cdot\left(T3_0 - T3_1\right) + \varepsilon1\cdot A2c5\cdot\sigma\cdot\left[\left(T2_1\right)^4 - \left(T3_1\right)^4\right]$$

$$h3\cdot A3c5\cdot\left(Tn - T3_2\right) - \frac{k4\cdot A3c5}{t2}\cdot\left(T3_2 - T4_2\right) - \frac{k4\cdot As3c5}{\left(\frac{D4c5 - D3c5}{2}\right)}\cdot\left(T3_2 - T3_3\right) \ldots = 0.0$$
$$+ \frac{k4\cdot As2c5}{\left(\frac{D3c5 - D2c5}{2}\right)}\cdot\left(T3_1 - T3_2\right) + \varepsilon1\cdot A3c5\cdot\sigma\cdot\left[\left(T2_2\right)^4 - \left(T3_2\right)^4\right]$$

$$h3\cdot A4c5\cdot\left(Tn - T3_3\right) - \frac{k4\cdot A4c5}{t2}\cdot\left(T3_3 - T4_3\right) - \frac{k4\cdot As4c5}{\left(\frac{D5c5 - D4c5}{2}\right)}\cdot\left(T3_3 - T3_4\right) \ldots = 0.0$$
$$+\frac{k4\cdot As3c5}{\left(\frac{D4c5 - D3c5}{2}\right)}\cdot\left(T3_2 - T3_3\right) + \varepsilon1\cdot A4c5\cdot\sigma\cdot\left[\left(T2_3\right)^4 - \left(T3_3\right)^4\right]$$

$$h3\cdot A5c5\cdot\left(Tn - T3_4\right) - \frac{k4\cdot A5c5}{t2}\cdot\left(T3_4 - T4_4\right) - \frac{k4\cdot As5c5}{\left(\frac{D6c5 - D5c5}{2}\right)}\cdot\left(T3_4 - T3_5\right) \ldots = 0.0$$
$$+\frac{k4\cdot As4c5}{\left(\frac{D5c5 - D4c5}{2}\right)}\cdot\left(T3_3 - T3_4\right) + \varepsilon1\cdot A5c5\cdot\sigma\cdot\left[\left(T2_4\right)^4 - \left(T3_4\right)^4\right]$$

$$h3\cdot A6c5\cdot\left(Tn - T3_5\right) - \frac{k4\cdot A6c5}{t2}\cdot\left(T3_5 - T4_5\right) - \frac{k4\cdot As6c5}{\left(\frac{D7c5 - D6c5}{2}\right)}\cdot\left(T3_5 - T3_6\right) \ldots = 0.0$$
$$+\frac{k4\cdot As5c5}{\left(\frac{D6c5 - D5c5}{2}\right)}\cdot\left(T3_4 - T3_5\right) + \varepsilon1\cdot A6c5\cdot\sigma\cdot\left[\left(T2_5\right)^4 - \left(T3_5\right)^4\right]$$

$$h3\cdot A7c5\cdot\left(Tn - T3_6\right) - \frac{k4\cdot A7c5}{t2}\cdot\left(T3_6 - T4_6\right) - \frac{k4\cdot As7c5}{\left(\frac{D8c5 - D7c5}{2}\right)}\cdot\left(T3_6 - T3_7\right) \ldots = 0.0$$
$$+\frac{k4\cdot As6c5}{\left(\frac{D7c5 - D6c5}{2}\right)}\cdot\left(T3_5 - T3_6\right) + \varepsilon1\cdot A7c5\cdot\sigma\cdot\left[\left(T2_6\right)^4 - \left(T3_6\right)^4\right]$$

$$h3\cdot A8c5\cdot\left(Tn - T3_7\right) - \frac{k4\cdot A8c5}{t2}\cdot\left(T3_7 - T4_7\right) + \frac{k4\cdot As7c5}{\left(\frac{D8c5 - D7c5}{2}\right)}\cdot\left(T3_6 - T3_7\right) \ldots = 0.0$$
$$+\frac{-k2\cdot As8c5}{t5}\cdot\left(T3_7 - Ts\right) + \varepsilon1\cdot A8c5\cdot\sigma\cdot\left[\left(T2_7\right)^4 - \left(T3_7\right)^4\right] - \varepsilon1\cdot\frac{\pi\cdot D8c5\cdot t2}{2}\cdot\sigma\cdot\left[\left(T3_7\right)^4 - (Ts)^4\right]$$

Thermal Equilibrium of Surface #4 of C5

$$\frac{k4\cdot A1c5}{t2}\cdot\left(T3_0 - T4_0\right) - \varepsilon1\cdot A1c5\cdot\sigma\cdot\left[\left(T4_0\right)^4 - T5^4\right] - \frac{k4\cdot As1c5}{\left(\frac{D2c5}{2}\right)}\cdot\left(T4_0 - T4_1\right) = 0.0$$

$$\frac{k4\cdot A2c5}{t2}\cdot\left(T3_1 - T4_1\right) - \varepsilon1\cdot A2c5\cdot\sigma\cdot\left[\left(T4_1\right)^4 - T5^4\right] + \frac{k4\cdot As1c5}{\left(\frac{D2c5}{2}\right)}\cdot\left(T4_0 - T4_1\right) - \frac{k4\cdot As2c5}{\left(\frac{D3c5 - D2c5}{2}\right)}\cdot\left(T4_1 - T4_2\right) = 0.0$$

$$\frac{k4\cdot A3c5}{t2}\cdot\left(T3_2 - T4_2\right) - \varepsilon1\cdot A3c5\cdot\sigma\cdot\left[\left(T4_2\right)^4 - T5^4\right] + \frac{k4\cdot As2c5}{\left(\frac{D3c5 - D2c5}{2}\right)}\cdot\left(T4_1 - T4_2\right) - \frac{k4\cdot As3c5}{\left(\frac{D4c5 - D3c5}{2}\right)}\cdot\left(T4_2 - T4_3\right) = 0.$$

$$\frac{k4\cdot A4c5}{t2}\cdot\left(T3_3 - T4_3\right) - \varepsilon1\cdot A4c5\cdot\sigma\cdot\left[\left(T4_3\right)^4 - T5^4\right] + \frac{k4\cdot As3c5}{\left(\frac{D4c5 - D3c5}{2}\right)}\cdot\left(T4_2 - T4_3\right) - \frac{k4\cdot As4c5}{\left(\frac{D5c5 - D4c5}{2}\right)}\cdot\left(T4_3 - T4_4\right) = 0.$$

$$\frac{k4\cdot A5c5}{t2}\cdot\left(T3_4 - T4_4\right) - \varepsilon1\cdot A5c5\cdot\sigma\cdot\left[\left(T4_4\right)^4 - T5^4\right] + \frac{k4\cdot As4c5}{\left(\frac{D5c5 - D4c5}{2}\right)}\cdot\left(T4_3 - T4_4\right) - \frac{k4\cdot As5c5}{\left(\frac{D6c5 - D5c5}{2}\right)}\cdot\left(T4_4 - T4_5\right) = 0.$$

$$\frac{k4\cdot A6c5}{t2}\cdot\left(T3_5 - T4_5\right) - \varepsilon1\cdot A6c5\cdot\sigma\cdot\left[\left(T4_5\right)^4 - T5^4\right] + \frac{k4\cdot As5c5}{\left(\frac{D6c5 - D5c5}{2}\right)}\cdot\left(T4_4 - T4_5\right) - \frac{k4\cdot As6c5}{\left(\frac{D7c5 - D6c5}{2}\right)}\cdot\left(T4_5 - T4_6\right) = 0.$$

$$\frac{k4\cdot A7c5}{t2}\cdot\left(T3_6 - T4_6\right) - \varepsilon1\cdot A7c5\cdot\sigma\cdot\left[\left(T4_6\right)^4 - T5^4\right] + \frac{k4\cdot As6c5}{\left(\frac{D7c5 - D6c5}{2}\right)}\cdot\left(T4_5 - T4_6\right) - \frac{k4\cdot As7c5}{\left(\frac{D8c5 - D7c5}{2}\right)}\cdot\left(T4_6 - T4_7\right) = 0.$$

$$\frac{k4\cdot A8c5}{t2}\cdot\left(T3_7 - T4_7\right) - \varepsilon1\cdot A8c5\cdot\sigma\cdot\left[\left(T4_7\right)^4 - T5^4\right] \ldots = 0.0$$
$$+ \frac{k4\cdot As7c5}{\left(\frac{D8c5 - D7c5}{2}\right)}\cdot\left(T4_6 - T4_7\right) - \frac{k2\cdot As8c5}{t5}\cdot\left(T4_7 - Ts\right) - \varepsilon1\cdot\frac{\pi\cdot D8c5\cdot t2}{2}\cdot\sigma\cdot\left[\left(T4_7\right)^4 - (Ts)^4\right]$$

$$\begin{pmatrix} T1a \\ T2a \\ T3a \\ T4a \\ Tna \end{pmatrix} := \mathrm{Find}(T1, T2, T3, T4, Tn)$$

$$T1a = \begin{pmatrix} 287.715 \\ 287.890 \\ 288.420 \\ 288.848 \\ 289.409 \\ 290.135 \\ 291.060 \\ 292.211 \end{pmatrix} K \qquad T2a = \begin{pmatrix} 286.026 \\ 286.205 \\ 286.749 \\ 287.193 \\ 287.781 \\ 288.554 \\ 289.565 \\ 290.886 \end{pmatrix} K \qquad T3a = \begin{pmatrix} 271.756 \\ 272.050 \\ 272.973 \\ 273.763 \\ 274.847 \\ 276.329 \\ 278.343 \\ 281.034 \end{pmatrix} K \qquad T4a = \begin{pmatrix} 262.465 \\ 262.787 \\ 263.806 \\ 264.686 \\ 265.909 \\ 267.619 \\ 270.037 \\ 273.518 \end{pmatrix} K \qquad Tna = 290.817\,K$$

$T1 := T1a$ $\quad$ $T2 := T2a$ $\quad$ $T3 := T3a$ $\quad$ $T4 := T4a$ $\quad$ $Tn := Tna$

$To := 2 \cdot Tn - Ti$ $\quad$ $To = 283.633\,K$ $\quad$ Outlet Temp of N2

$Ti = 298.000\,K$ $\quad$ Given Inlet Temp. $\quad$ $Ti - To = 14.367\,K$ $\quad$ $\frac{Ti}{To} = 1.051$

$$T3a - T4a = \begin{pmatrix} 9.291 \\ 9.262 \\ 9.167 \\ 9.076 \\ 8.937 \\ 8.710 \\ 8.306 \\ 7.516 \end{pmatrix} K$$

1.5.0 Calculation of Heat Flux

Calculation of Heat Flux into C5

Calculation of heat flux to C5 from radiation

$$\begin{aligned} &\varepsilon1 \cdot A1c5 \cdot \sigma \cdot \left[\left(T4_0\right)^4 - T5^4\right] + \varepsilon1 \cdot A2c5 \cdot \sigma \cdot \left[\left(T4_1\right)^4 - T5^4\right] + \varepsilon1 \cdot A3c5 \cdot \sigma \cdot \left[\left(T4_2\right)^4 - T5^4\right] \ldots \\ &+ \varepsilon1 \cdot A4c5 \cdot \sigma \cdot \left[\left(T4_3\right)^4 - T5^4\right] + \varepsilon1 \cdot A5c5 \cdot \sigma \cdot \left[\left(T4_4\right)^4 - T5^4\right] + \varepsilon1 \cdot A6c5 \cdot \sigma \cdot \left[\left(T4_5\right)^4 - T5^4\right] \ldots \\ &+ \varepsilon1 \cdot A7c5 \cdot \sigma \cdot \left[\left(T4_6\right)^4 - T5^4\right] + \varepsilon1 \cdot A8c5 \cdot \sigma \cdot \left[\left(T4_7\right)^4 - T5^4\right] \end{aligned} = 46.357\,W$$

Heat Flux from convection on surface T3 into C5

$$\begin{aligned} &h3 \cdot A1c5 \cdot \left(T3_0 - Tn\right) + h3 \cdot A2c5 \cdot \left(T3_1 - Tn\right) \ldots \\ &+ h3 \cdot A3c5 \cdot \left(T3_2 - Tn\right) + h3 \cdot A4c5 \cdot \left(T3_3 - Tn\right) + h3 \cdot A5c5 \cdot \left(T3_4 - Tn\right) + h3 \cdot A6c5 \cdot \left(T3_5 - Tn\right) \ldots \\ &+ h3 \cdot A7c5 \cdot \left(T3_6 - Tn\right) + h3 \cdot A8c5 \cdot \left(T3_7 - Tn\right) \end{aligned} = -20.368\,W$$

Heat Flux by Conductivity through outer perimeter of C5

$$\frac{k2\cdot As8c5\cdot\left(T3_7 - Ts\right)}{t5} + \frac{k2\cdot As8c5\cdot\left(T4_7 - Ts\right)}{t5} = -4.587\ W$$

Calculation of Heat Flux from Radiation from T2 Surface of C4 to T3 Surface of C5

$$\begin{aligned}
&\varepsilon1\cdot A1c5\cdot\sigma\cdot\left[\left(T2_0\right)^4 - \left(T3_0\right)^4\right] + \varepsilon1\cdot A2c5\cdot\sigma\cdot\left[\left(T2_1\right)^4 - \left(T3_1\right)^4\right] + \varepsilon1\cdot A3c5\cdot\sigma\cdot\left[\left(T2_2\right)^4 - \left(T3_2\right)^4\right] \ldots = 12.501\ W\\
&+ \varepsilon1\cdot A4c5\cdot\sigma\cdot\left[\left(T2_3\right)^4 - \left(T3_3\right)^4\right] \ldots\\
&+ \varepsilon1\cdot A5c5\cdot\sigma\cdot\left[\left(T2_4\right)^4 - \left(T3_4\right)^4\right] + \varepsilon1\cdot A6c5\cdot\sigma\cdot\left[\left(T2_5\right)^4 - \left(T3_5\right)^4\right] \ldots\\
&+ \varepsilon1\cdot A7c5\cdot\sigma\cdot\left[\left(T2_6\right)^4 - \left(T3_6\right)^4\right] + \varepsilon1\cdot A8c5\cdot\sigma\cdot\left[\left(T2_7\right)^4 - \left(T3_7\right)^4\right]
\end{aligned}$$

Calculation of Radiation Heat Flux at the Outer Radius of C5

$$\varepsilon1\cdot\frac{\pi\cdot D8c5\cdot t2}{2}\cdot\sigma\cdot\left[\left(T3_7\right)^4 - (Ts)^4\right] + \varepsilon1\cdot\frac{\pi\cdot D8c5\cdot t2}{2}\cdot\sigma\cdot\left[\left(T4_7\right)^4 - (Ts)^4\right] = -8.901\ W$$

Calculation of Heat Flux into N2 Region

Heat flux from C4

$$\begin{aligned}
&h2\cdot A1c4\cdot\left(T2_0 - Tn\right) + h2\cdot A2c4\cdot\left(T2_1 - Tn\right) + h2\cdot A3c4\cdot\left(T2_2 - Tn\right) \ldots = -3.418\ W\\
&+ h2\cdot A4c4\cdot\left(T2_3 - Tn\right) + h2\cdot A5c4\cdot\left(T2_4 - Tn\right) \ldots\\
&+ h2\cdot A6c4\cdot\left(T2_5 - Tn\right) + h2\cdot A7c4\cdot\left(T2_6 - Tn\right) + h2\cdot A8c4\cdot\left(T2_7 - Tn\right)
\end{aligned}$$

Heat Flux from C5

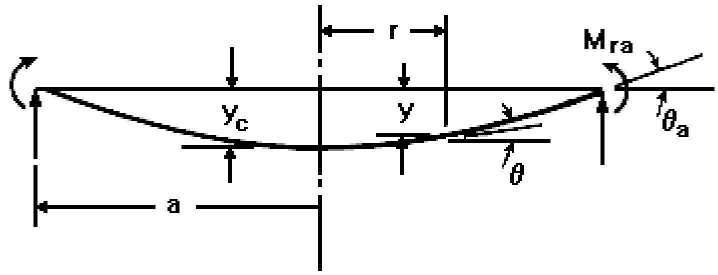
$$\begin{aligned}
&h3\cdot A1c5\cdot\left(T3_0 - Tn\right) + h3\cdot A2c5\cdot\left(T3_1 - Tn\right) \ldots = -20.368\ W\\
&+ h3\cdot A3c5\cdot\left(T3_2 - Tn\right) + h3\cdot A4c5\cdot\left(T3_3 - Tn\right) + h3\cdot A5c5\cdot\left(T3_4 - Tn\right) + h3\cdot A6c5\cdot\left(T3_5 - Tn\right) \ldots\\
&+ h3\cdot A7c5\cdot\left(T3_6 - Tn\right) + h3\cdot A8c5\cdot\left(T3_7 - Tn\right)
\end{aligned}$$

Heat Absorption of N2

$$mN2\cdot cpN2\cdot 2\cdot(Ti - Tn) = 23.786\ W$$

2.0 Calculation of C5 Stresses and Deflections Due to Temperature Differences

Solid circular plate



Uniform temperature differential between the bottom and top surface from r_o to a

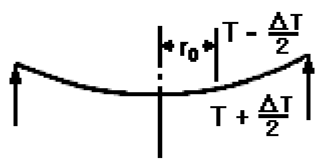


Plate dimensions:

thickness: $t \equiv 55.1mm$

radius: $a \equiv 271 \cdot mm$

Temperature differential (°F): $\Delta T \equiv -9.18$ Based on Calculations in previous section

Temperature coefficient of expansion ($°F^{-1}$): $\gamma \equiv .305 \cdot 10^{-6} \cdot \frac{in}{in}$

Modulus of elasticity: $E \equiv 10544243.5 \cdot \frac{lbf}{in^2}$

Poisson's ratio: $\nu \equiv 0.16$

Radial location of applied load: $r_o \equiv 0.00001 \cdot in$

D is a plate constant used in determining boundary values; it is also used in the general equations for deflection, slope, moment and shear.

$$D \equiv \frac{E\cdot t^3}{12\cdot\left(1 - \nu^2\right)}$$ $$D = 9205567.185\ \text{lbf}\cdot\text{in}$$

Boundary values

The L_n functions used in the equations below are defined at the end of this document.

M_r is radial moment, Q is shear, y is deflection and θ is slope.

Due to bending:

At the edge of the plate (a):

$$M_{raTemp} := 0\cdot\frac{\text{lbf}\cdot\text{in}}{\text{in}}$$ $$M_{raTemp} = 0.000\ \frac{\text{lbf}\cdot\text{in}}{\text{in}}$$

$$y_{aTemp} := 0\cdot\text{in}$$ $$y_{aTemp} = 0.000\ \text{in}$$

$$\theta_{aTemp} := \frac{\gamma\cdot\Delta T}{t\cdot a}\cdot\left(a^2 - r_o^{\,2}\right)$$ $$\theta_{aTemp} = -0.001\ \text{deg}$$

$$Q_{aTemp} \equiv 0\cdot\frac{\text{lbf}}{\text{in}}$$ $$Q_{aTemp} = 0.000\ \frac{\text{lbf}}{\text{in}}$$

At the center of the plate (c):

$$M_{cTemp} := \frac{\gamma\cdot D\cdot(1 + \nu)\cdot\Delta T}{t}\cdot\left(1 - L_8\right)$$ $$M_{cTemp} = -5.789\ \frac{\text{lbf}\cdot\text{in}}{\text{in}}$$

$$y_{cTemp} := \frac{-\gamma\cdot\Delta T}{2\cdot t}\cdot\left[a^2 - r_o^{\,2} - r_o^{\,2}\cdot(1 + \nu)\cdot\ln\left(\frac{a}{r_o}\right)\right]$$ $$y_{cTemp} = 0.00007\ \text{in}$$

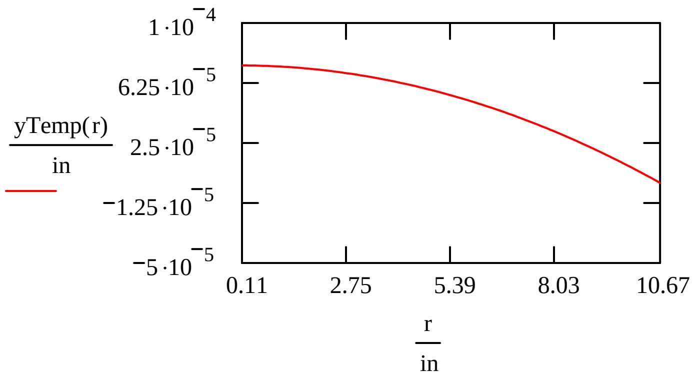
General formulas and graphs for deflection, slope, moment, shear and stress as a function of r

Define r, the range of the radius:

$$r \equiv \frac{a}{100}, \frac{a}{50} .. a$$

Deflection

$$yTemp(r) := y_{cTemp} + \frac{M_{cTemp} \cdot r^2}{2 \cdot D \cdot (1 + \nu)} + LT_y(r)$$



Deflections at the center and outer radius:

$yTemp(0 \cdot in) = 0.00007\ in$ $\quad$ $yTemp(a) = 0.00000\ in$

Maximum deflection (magnitude):

$Y_{(r) \cdot \frac{100}{mm}} := yTemp(r)$ $\quad$ $A := max(Y)$ $\quad$ $B := min(Y)$

$y_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\quad$ $y_{max} = 7.346 \times 10^{-5}\ in$

Large deflection condition check

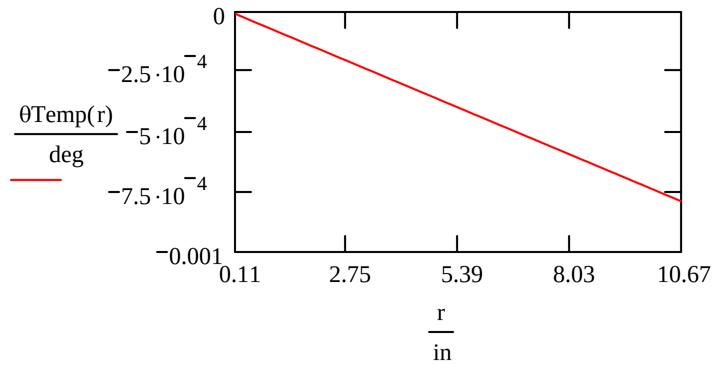
Check to verify that the absolute value of the maximum deflection is less than one-half the plate thickness (an assumption stated in the Notation file which must hold true):

$check := if\left(\left|y_{max}\right| > \frac{t}{2}, 0, 1\right)$ $\quad$ $check = 1.000$

If $|y_{max}|$ is greater than t/2 (i.e., check = 0), the equations in this table are subject to large errors. For large deflections, use the equations provided in Table 24a to obtain stress and deflection.

Slope

$$\theta Temp(r) := \frac{M_{cTemp} \cdot r}{D \cdot (1 + \nu)} + LT_\theta(r)$$



Slope at center and outer radius:

$\theta Temp(0 \cdot in) = 0.000\,deg$

$\theta Temp(a) = -7.890 \times 10^{-4}\,deg$

Maximum slope (magnitude):

$S_{(r) \cdot \frac{100}{mm}} := \theta Temp(r)$ $\quad A := \max(S)$ $\quad B := \min(S)$

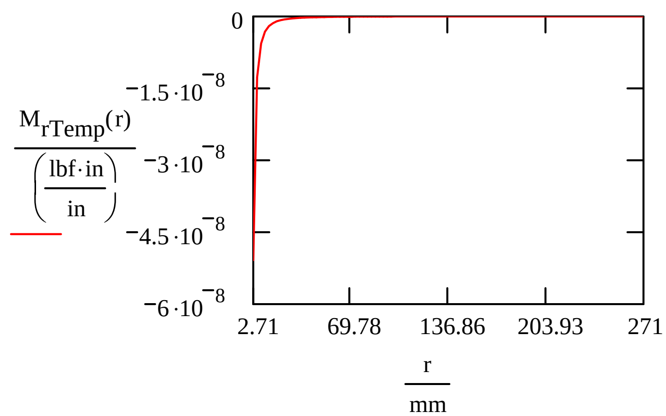
$\theta_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$

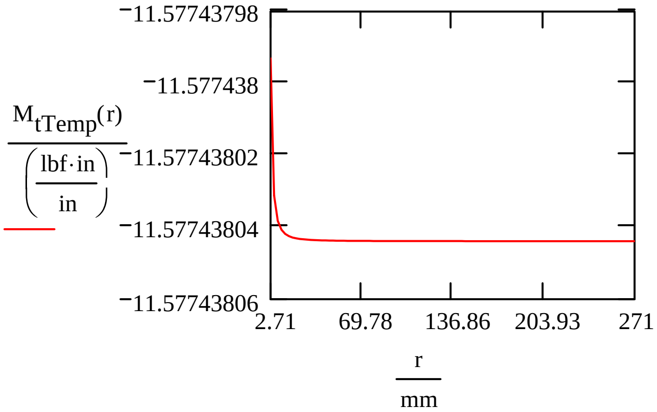
$\theta_{max} = -7.890 \times 10^{-4}\,deg$

Moment; radial and tangential

$M_{rTemp}(r) := M_{cTemp} + LT_M(r)$

$$M_{tTemp}(r) := \frac{\theta Temp(r) \cdot D \cdot \left(1 - \nu^2\right)}{r} + \nu \cdot M_{rTemp}(r)$$





Radial and tangential moment near center and outer radii:

$M_{rTemp}(0.00001\cdot in) = -5.789\,\frac{lbf\cdot in}{in}$ $\quad$ $M_{rTemp}(a) = 0.000\,\frac{lbf\cdot in}{in}$

$M_{tTemp}(0.0001\cdot in) = -11.520\,\frac{lbf\cdot in}{in}$ $\quad$ $M_{tTemp}(a) = -11.577\,\frac{lbf\cdot in}{in}$

Maximum radial and tangential moment (magnitude):

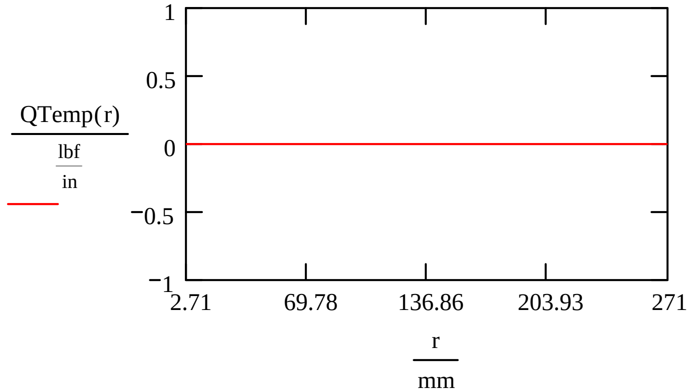
$Mr_{(r)\cdot\frac{100}{mm}} := M_{rTemp}(r)$ $\quad Ar := \max(Mr)$ $\quad B := \min(Mr)$

$Mt_{(r)\cdot\frac{100}{mm}} := M_{tTemp}(r)$ $\quad At := \max(Mt)$ $\quad Bt := \min(Mt)$

$Mr_{max} := (Ar > -B)\cdot Ar + (Ar \le -B)\cdot B$ $\quad$ $Mr_{max} = -0.000\,\frac{lbf\cdot in}{in}$

$Mt_{max} := (At > -Bt)\cdot At + (At \le -Bt)\cdot Bt$ $\quad$ $Mt_{max} = -11.577\,\frac{lbf\cdot in}{in}$

Shear $\quad QTemp(r) := LT_Q(r)$



Shear at center and outer radius:

$QTemp(0.01\cdot in) = 0.000\,\frac{lbf}{in}$ $\qquad$ $QTemp(a) = 0.000\,\frac{lbf}{in}$

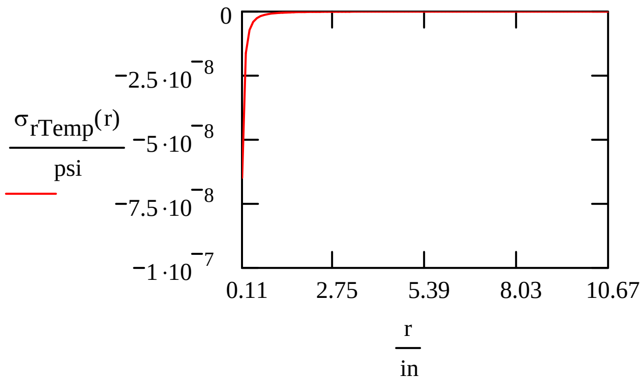
Maximum shear (magnitude):

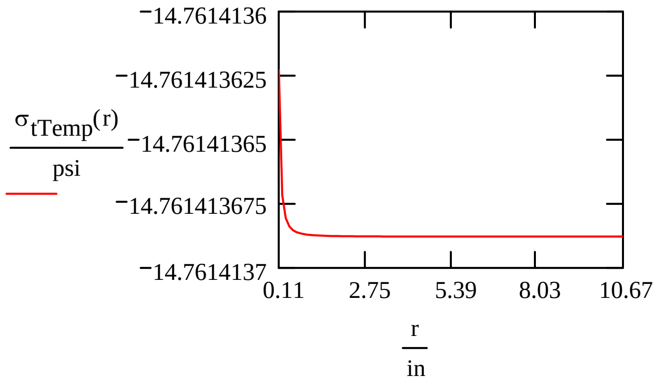
$V_{(r)\cdot\frac{100}{mm}} := QTemp(r) \quad A := max(V) \qquad B := min(V)$

$Q_{max} := (A > -B)\cdot A + (A \le -B)\cdot B \qquad Q_{max} = 0.000\,\frac{lbf}{in}$

Bending stresses; radial and tangential

$$\sigma_{rTemp}(r) := \frac{6\cdot M_{rTemp}(r)}{t^2} \qquad \sigma_{tTemp}(r) := \frac{6\cdot M_{tTemp}(r)}{t^2}$$





Radial and tangential stress at center and outer radius:

$\sigma_{rTemp}(0.0001\cdot in) = -0.074\,psi$ $\qquad$ $\sigma_{rTemp}(a) = 0.000\,psi$

$\sigma_{tTemp}(0.0001\cdot in) = -14.688\,psi$ $\qquad$ $\sigma_{tTemp}(a) = -14.761\,psi$

Maximum radial and tangential stresses:

$\sigma r_{r\cdot\frac{100}{mm}} := \sigma_{rTemp}(r)$ $\quad$ $Ar := \max(\sigma r)$ $\qquad$ $Br := \min(\sigma r)$

$\sigma t_{r\cdot\frac{100}{mm}} := \sigma_{tTemp}(r)$ $\quad$ $At := \max(\sigma t)$ $\qquad$ $Bt := \min(\sigma t)$

$\sigma r_{max} := (Ar > -Br)\cdot Ar + (Ar \le -Br)\cdot Br$ $\qquad$ $\sigma r_{max} = -6.483 \times 10^{-8}\,psi$

$\sigma t_{max} := (At > -Bt)\cdot At + (At \le -Bt)\cdot Bt$ $\qquad$ $\sigma t_{max} = -14.761\,psi$

Review the maximum values for deflection, slope, moment, stress and shear

$y_{max} = 7.346 \times 10^{-5}\,in$ $\qquad$ $\theta_{max} = -0.001\,deg$

$Mr_{max} = -0.000\,\frac{lbf\cdot in}{in}$ $\qquad$ $Mt_{max} = -11.577\,\frac{lbf\cdot in}{in}$

$\sigma r_{max} = -0.000\,psi$ $\qquad$ $\sigma t_{max} = -14.761\,psi$

$Q_{max} = 0.000\,\frac{lbf}{in}$

$$L_8 \equiv \frac{1}{2}\cdot\left[1+\nu+(1-\nu)\cdot\left(\frac{r_o}{a}\right)^2\right]$$

$$G_2(r) \equiv \text{if}\left[r>r_o, \frac{1}{4}\cdot\left[1-\left(\frac{r_o}{r}\right)^2\cdot\left(1+2\cdot\ln\left(\frac{r}{r_o}\right)\right)\right], 0\right]$$

$$G_5(r) \equiv \text{if}\left[r>r_o, \frac{1}{2}\cdot\left[1-\left(\frac{r_o}{r}\right)^2\right], 0\right]$$

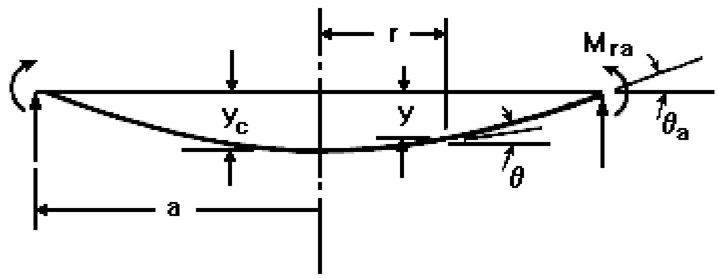
$$G_8(r) \equiv \text{if}\left[r>r_o, \frac{1}{2}\cdot\left[1+\nu+(1-\nu)\cdot\left(\frac{r_o}{r}\right)^2\right], 0\right]$$

$$LT_y(r) \equiv \frac{\gamma\cdot(1+\nu)\cdot\Delta T}{t}\cdot r^2\cdot G_2(r) \qquad LT_\theta(r) \equiv \frac{\gamma\cdot(1+\nu)\cdot\Delta T}{t}\cdot r\cdot G_5(r)$$

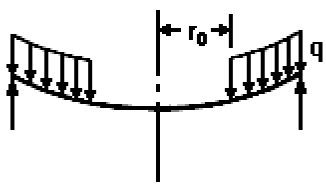
$$LT_M(r) \equiv \frac{\gamma\cdot D\cdot(1+\nu)\cdot\Delta T}{t}\cdot\left[G_8(r)-(r>r_o)\right] \qquad LT_Q(r) \equiv 0\cdot\frac{lbf}{in}$$

3.0 Calculation of C5 Stresses and Deflections Due to 1atm Pressure

Solid circular plate



Uniformly distributed pressure from r_o to a



Enter dimensions, properties and loading

Plate dimensions:

thickness: $t \equiv 55.1mm$

radius: $a \equiv 271 \cdot mm$

Applied uniform pressure: $q \equiv 14 \cdot psi$

Modulus of elasticity: $E \equiv 9.999 \cdot 10^6 \cdot \frac{lbf}{in^2}$

Poisson's ratio: $\nu \equiv 0.16$

Radial location of applied load: $r_o \equiv 0.00001 \cdot in$

Shear modulus: $G \equiv \frac{E}{2 \cdot (1 + \nu)}$

D is a plate constant used in determining boundary values; it is also used in the general equations for deflection, slope, moment and shear. K_{sro} is the tangential shear constant used in determining the deflection due to shear.

$$D \equiv \frac{E \cdot t^3}{12 \cdot \left(1 - \nu^2\right)} \qquad D = 8729546.722\ \mathrm{lbf \cdot in}$$

$$K_{sro} \equiv \mathrm{if}\left[r_o > 0, -0.30 \cdot \left[1 - \left(\frac{r_o}{a}\right)^2 \cdot \left(1 + 2 \cdot \ln\left(\frac{a}{r_o}\right)\right)\right], -0.30\right] \qquad K_{sro} = -0.300$$

Boundary values

The G_n and L_n functions used in the equations below are defined at the end of this document.

M_r is radial moment, Q is shear, y is deflection and θ is slope.

Due to bending:

At the edge of the plate (a):

$$M_{raP} := 0 \cdot \frac{\mathrm{lbf \cdot in}}{\mathrm{in}} \qquad M_{raP} = 0.000\ \frac{\mathrm{lbf \cdot in}}{\mathrm{in}}$$

$$Q_{aP} := \frac{-q}{2 \cdot a} \cdot \left(a^2 - r_o^2\right) \qquad Q_{aP} = -74.685\ \frac{\mathrm{lbf}}{\mathrm{in}}$$

$$y_a := 0 \cdot \mathrm{in} \qquad y_a = 0.000\ \mathrm{in}$$

$$\theta_{aP} := \frac{q}{8 \cdot D \cdot a \cdot (1 + \nu)} \cdot \left(a^2 - r_o^2\right)^2 \qquad \theta_{aP} = 0.012\ \mathrm{deg}$$

At the center of the plate (c):

$$M_{cP} := q \cdot a^2 \cdot L_{17} \qquad M_{cP} = 314.750\ \frac{\mathrm{lbf \cdot in}}{\mathrm{in}}$$

$$y_{cP} := \frac{-q \cdot a^4}{2 \cdot D} \cdot \left(\frac{L_{17}}{1 + \nu} - 2 \cdot L_{11}\right) \qquad y_{cP} = -0.001\ \mathrm{in}$$

Due to tangential shear stresses:

$$y_{sroP} := \frac{K_{sro} \cdot q \cdot a^2}{t \cdot G} \qquad y_{sroP} = -5.1137 \times 10^{-5}\ \mathrm{in}$$

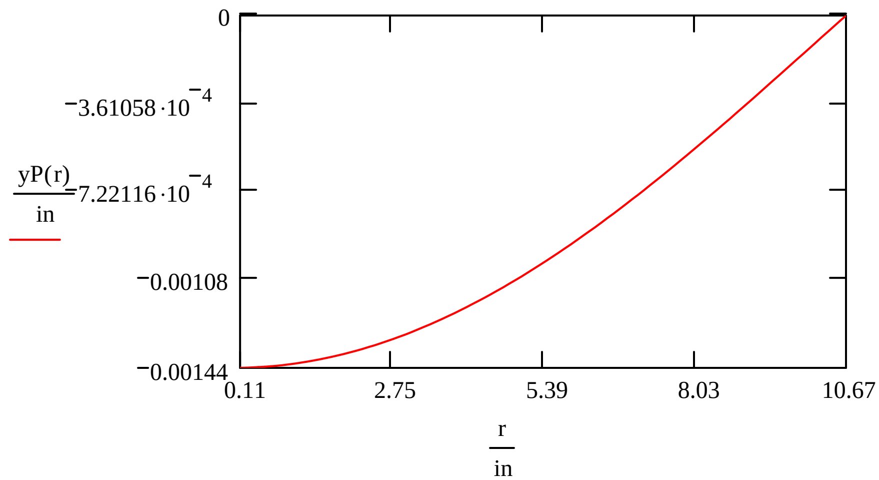
General formulas and graphs for deflection, slope, moment, shear and stress as a function of r

Define r, the range of the radius:

$$r \equiv \frac{a}{100}, \frac{a}{50} .. a$$

Deflection

$$yP(r) := y_{cP} + \frac{M_{cP} \cdot r^2}{2 \cdot D \cdot (1 + \nu)} + LT_{yP}(r)$$



Deflections at the center and outer radius:

$yP(0 \cdot in) = -0.001\ in$ $\qquad$ $yP(a) = 0.000\ in$

Maximum deflection (magnitude):

$Y_{(r) \cdot \frac{100}{a}} := yP(r)$ $\qquad$ $A := max(Y)$ $\qquad$ $B := min(Y)$

$y_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\qquad$ $y_{max} = -0.0014\ in$

Large deflection condition check

Check to verify that the absolute value of the maximum deflection is less than one-half the plate thickness (an assumption stated in the

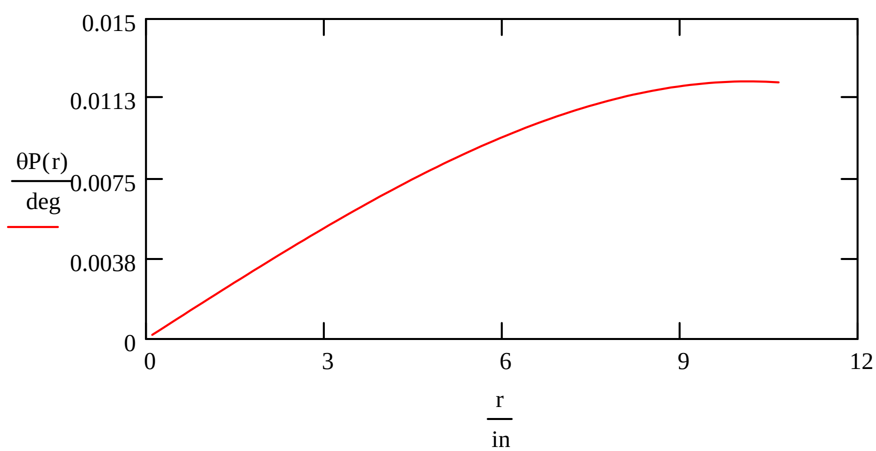
Notation file which must hold true). If $|y_{max}|$ is greater than t/2 (large deflection), the equations in this table used for plates with small deflections are subject to large errors.

$$check := if\left(|y_{max}| > \frac{t}{2}, 0, 1\right) \qquad check = 1.000$$

If $|y_{max}|$ is greater than t/2 (i.e., check = 0), only max y and σ can be found.

Slope

$$\theta P(r) := \frac{M_{cP} \cdot r}{D \cdot (1 + \nu)} + LT_{\theta P}(r)$$



Slope at center and outer radius:

$$\theta P(0 \cdot in) = 0.000\ deg \qquad \theta P(a) = 0.012\ deg$$

Maximum slope (magnitude):

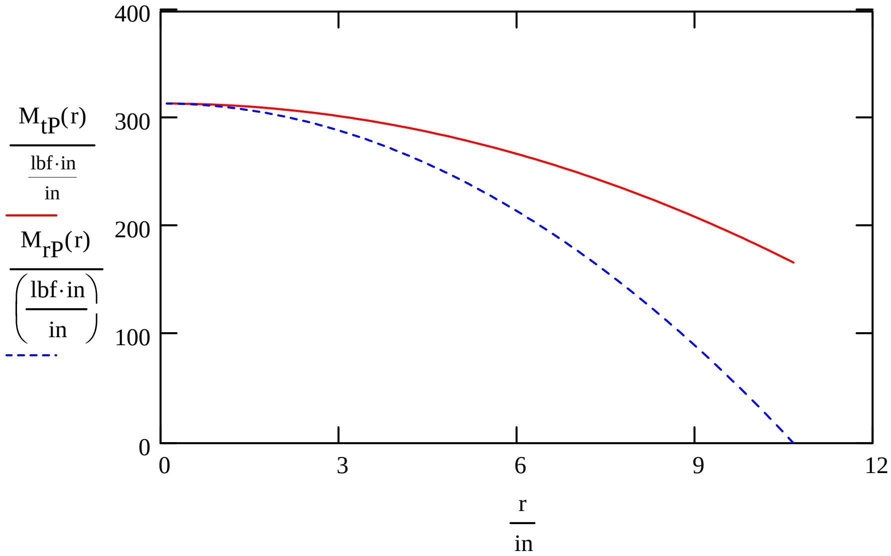
$$S_{(r) \cdot \frac{100}{a}} := \theta P(r) \qquad A := max(S) \qquad B := min(S)$$

$$\theta_{maxP} := (A > -B) \cdot A + (A \le -B) \cdot B \qquad \theta_{maxP} = 0.012\ deg$$

Moment; radial and tangential

$$M_{rP}(r) := M_{cP} + LT_{MP}(r)$$

$$M_{tP}(r) := \frac{\theta P(r) \cdot D \cdot \left(1 - \nu^2\right)}{r} + \nu \cdot M_{rP}(r)$$



Radial and tangential moment at center and outer radius:

$M_{rP}(0 \cdot in) = 314.750\ \frac{lbf \cdot in}{in}$ $\quad$ $M_{rP}(a) = 0.000\ \frac{lbf \cdot in}{in}$

$M_{tP}(0.01 \cdot in) = 314.750\ \frac{lbf \cdot in}{in}$ $\quad$ $M_{tP}(a) = 167.336\ \frac{lbf \cdot in}{in}$

Maximum radial and tangential moment (magnitude):

$Mr_{(r) \cdot \frac{100}{a}} := M_{rP}(r)$ $\quad$ $Ar := max(Mr)$ $\quad$ $B := min(Mr)$

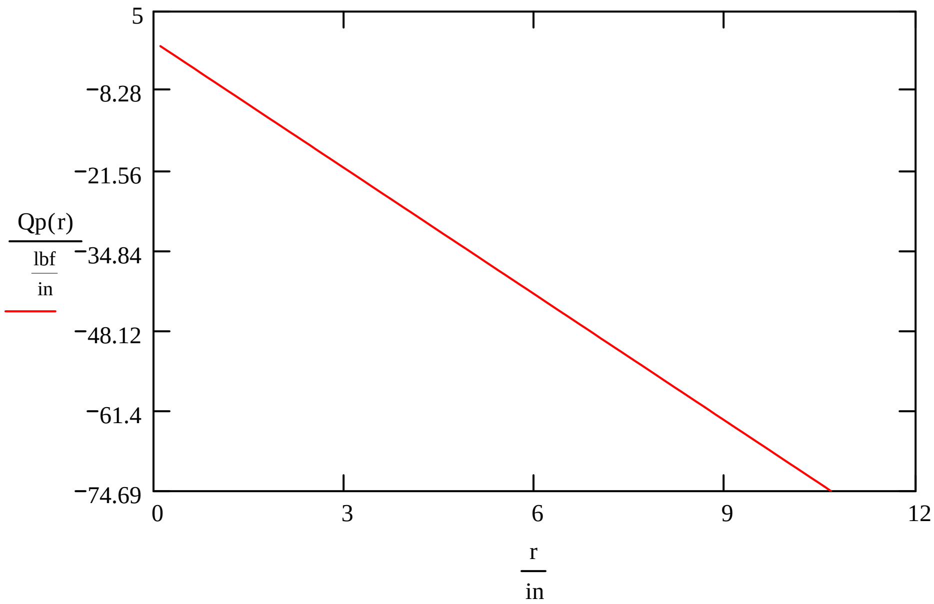
$Mt_{(r) \cdot \frac{100}{a}} := M_{tP}(r)$ $\quad$ $At := max(Mt)$ $\quad$ $Bt := min(Mt)$

$Mr_{max} := (Ar > -B) \cdot Ar + (Ar \le -B) \cdot B$ $\quad$ $Mr_{max} = 314.719\ \frac{lbf \cdot in}{in}$

$Mt_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\quad$ $Mt_{max} = 314.74\ \frac{lbf \cdot in}{in}$

Shear

$Qp(r) := LT_{QP}(r)$



Shear at center and outer radius:

$Qp(0.01 \cdot in) = -0.070 \, \frac{lbf}{in}$ $\qquad$ $Qp(a) = -74.685 \, \frac{lbf}{in}$

Maximum shear (magnitude):

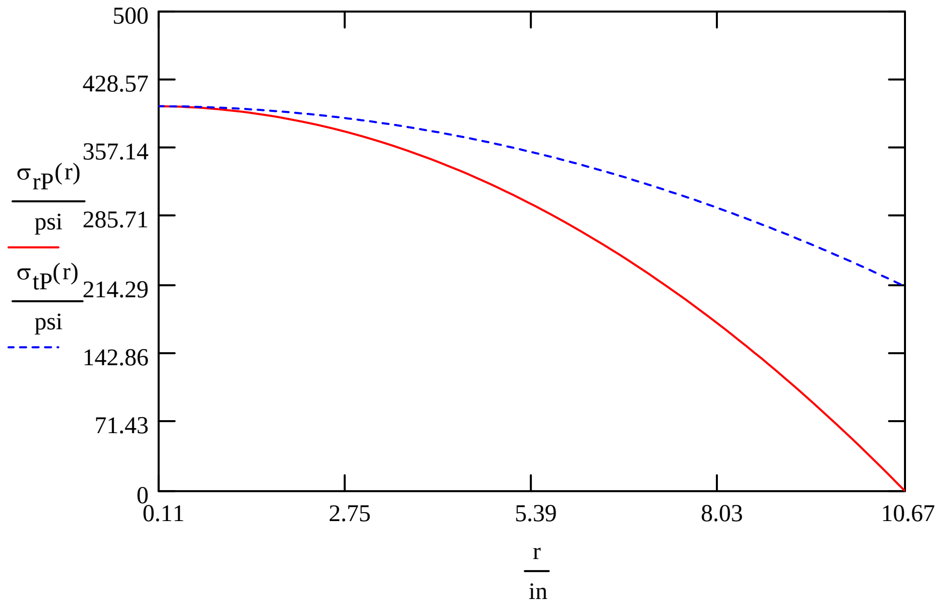
$V_{(r) \cdot \frac{100}{a}} := Qp(r)$ $\qquad$ $A := \max(V)$ $\qquad$ $B := \min(V)$

$Q_{max} := (A > -B) \cdot A + (A \le -B) \cdot B$ $\qquad$ $Q_{max} = -74.685 \, \frac{lbf}{in}$

Bending stresses; radial and tangential

$$\sigma_{rP}(r) := \frac{6 \cdot M_{rP}(r)}{t^2}$$

$$\sigma_{tP}(r) := \frac{6 \cdot M_{tP}(r)}{t^2}$$



Radial and tangential stress at center and outer radius:

$\sigma_{rP}(0.01 \cdot in) = 401.311\ psi$ $\qquad \sigma_{rP}(a) = 0.000\ psi$

$\sigma_{tP}(0.01 \cdot in) = 401.311\ psi$ $\qquad \sigma_{tP}(a) = 213.356\ psi$

Maximum radial and tangential stresses:

$\sigma r_{r \cdot \frac{100}{a}} := \sigma_{rP}(r)$ $\qquad Ar := \max(\sigma r)$ $\qquad Br := \min(\sigma r)$

$\sigma t_{r \cdot \frac{100}{a}} := \sigma_{tP}(r)$ $\qquad At := \max(\sigma t)$ $\qquad Bt := \min(\sigma t)$

$\sigma r_{max} := (Ar > -Br) \cdot Ar + (Ar \le -Br) \cdot Br$ $\qquad \sigma r_{max} = 401.272\ psi$

$\sigma t_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\qquad \sigma t_{max} = 401.293\ psi$

Review the maximum values for deflection, slope, moment, stress and shear

$y_{max} = -0.001\ in$ $\qquad \theta_{max} = -0.001\ deg$

$Mr_{max} = 314.719 \frac{lbf \cdot in}{in}$ $\qquad$ $Mt_{max} = 314.736 \frac{lbf \cdot in}{in}$

$\sigma r_{max} = 401.272\, psi$ $\qquad$ $\sigma t_{max} = 401.293\, psi$

$Q_{max} = -74.685 \frac{lbf}{in}$

Total deflection of plate (bending induced plus shear induced):

$y_{ro.total} := yP(0 \cdot in) + y_{sroP}$ $\qquad$ $y_{ro.total} = -0.0015\, in$

The remainder of the document displays the general plate functions and constants used in the equations above.

$$L_{11} \equiv if\left[r_o > 0, \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{a}\right)^2 - 5 \cdot \left(\frac{r_o}{a}\right)^4 \cdots + -4 \cdot \left(\frac{r_o}{a}\right)^2 \cdot \left[2 + \left(\frac{r_o}{a}\right)^2\right] \cdot \ln\left(\frac{a}{r_o}\right)\right], \frac{1}{64}\right]$$

$$L_{17} \equiv if\left[r_o > 0, \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{a}\right)^4\right] - \left(\frac{r_o}{a}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{a}{r_o}\right)\right]\right], \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right)\right]\right]$$

$$G_{11}(r) \equiv if\left[(r > r_o) \cdot (r_o > 0), \frac{1}{64} \cdot \left[1 + 4 \cdot \left(\frac{r_o}{r}\right)^2 - 5 \cdot \left(\frac{r_o}{r}\right)^4 + 4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \left[2 + \left(\frac{r_o}{r}\right)^2\right] \cdot \ln\left(\frac{r_o}{r}\right)\right], 0\right]$$

$$G_{14}(r) \equiv if\left[(r > r_o) \cdot (r_o > 0), \frac{1}{16} \cdot \left[1 - \left(\frac{r_o}{r}\right)^4 - 4 \cdot \left(\frac{r_o}{r}\right)^2 \cdot \ln\left(\frac{r_o}{r}\right)\right], 0\right]$$

$$G_{17}(r) \equiv if\left[\left[(r > r_o) \cdot (r > 0)\right], \frac{1}{4} \cdot \left[1 - \left(\frac{1-\nu}{4}\right) \cdot \left[1 - \left(\frac{r_o}{r}\right)^4\right] - \left(\frac{r_o}{r}\right)^2 \cdot \left[1 + (1+\nu) \cdot \ln\left(\frac{r}{r_o}\right)\right]\right], 0\right]$$

$G_{17}(10in) = 0.197$

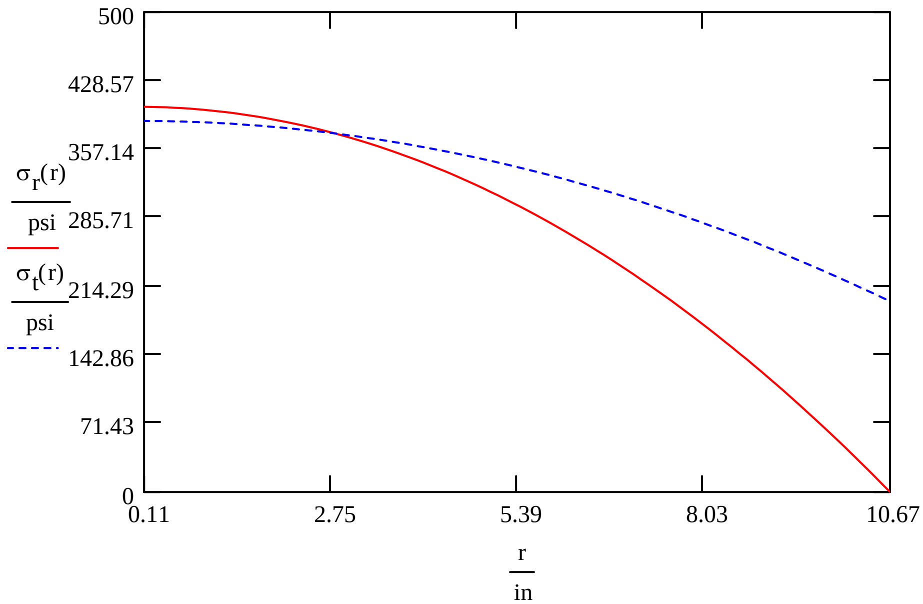
$LT_{yP}(r) \equiv \frac{-q \cdot r^4}{D} \cdot G_{11}(r)$ $\qquad$ $LT_{MP}(r) \equiv -q \cdot r^2 \cdot G_{17}(r)$

$LT_{\theta P}(r) \equiv \frac{-q \cdot r^3}{D} \cdot G_{14}(r)$ $\qquad$ $LT_{QP}(r) \equiv if\left[r > r_o, \frac{-q}{2 \cdot r} \cdot \left(r^2 - r_o^2\right), 0\right]$

4.0 Calculation of C5 Combined Stresses and Deflections Due to Pressure and Temperature Difference

$\sigma_r(r) := \sigma_{rP}(r) + \sigma_{rTemp}(r)$

$\sigma_t(r) := \sigma_{tP}(r) + \sigma_{tTemp}(r)$



Radial and tangential stress at center and outer radius:

$\sigma_r(0.01 \cdot in) = 401.311\ psi$ $\qquad$ $\sigma_{rP}(a) = 0.000\ psi$

$\sigma_t(0.01 \cdot in) = 386.550\ psi$ $\qquad$ $\sigma_{tP}(a) = 213.356\ psi$

Maximum radial and tangential stresses:

$\sigma r_{r \cdot \frac{100}{a}} := \sigma_r(r)$ $\qquad$ $Ar := \max(\sigma r)$ $\qquad$ $Br := \min(\sigma r)$

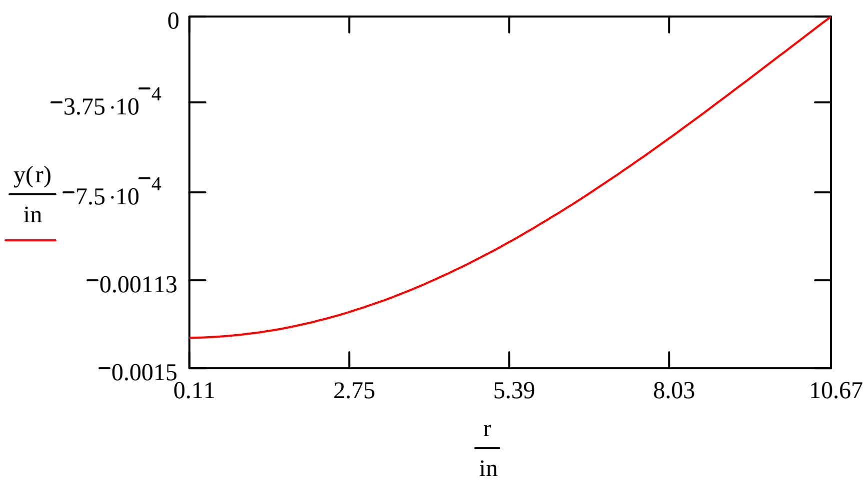
$\sigma t_{r \cdot \frac{100}{a}} := \sigma_t(r)$ $\qquad$ $At := \max(\sigma t)$ $\qquad$ $Bt := \min(\sigma t)$

$\sigma r_{max} := (Ar > -Br) \cdot Ar + (Ar \le -Br) \cdot Br$ $\qquad$ $\sigma r_{max} = 401.272\ psi$

$\sigma t_{max} := (At > -Bt) \cdot At + (At \le -Bt) \cdot Bt$ $\qquad$ $\sigma t_{max} = 386.531\ psi$

Deflections

$y(r) := yP(r) + yTemp(r)$



Deflections at the center and outer radius:

$y(0\cdot in) = -0.00137\ in$

$y(a) = 0.00000\ in$

Maximum deflection (magnitude):

$Y_{(r)\cdot\frac{100}{a}} := y(r)$ $\quad A := \max(Y)$ $\quad B := \min(Y)$

$y_{max} := (A > -B)\cdot A + (A \le -B)\cdot B$ $\quad y_{max} = -0.00137\ in$

5.0Comparison with FEA Results

An FEA model was created of the C5 lens. Radiation was applied to the T4 surface and Radiation and Convection was applied to the T3 Surface. Radiation heat transfer was also applied to the outer radius to the support structure. The convection coefficient calculated above was used in the model as well as the average temperature of the N2 and T2 surface of C4. The model was restrained by 2 o-rings at the outer radius on the T4 surface and 14psi was applied to the T3 surface.

The Figure below shows the stress distribution for the C5 Lens when simply supported at its edges and a 14psi pressure applied. The vonMises stresses are plotted which is a combined stress of the radial, tangential, and bearing stresses on the lens. The analysis above does not take into account the bearing stresses on the lens in the region of the support and only shows the stresses in the radial and tangential direcition separately. However, it can be seen that the scale of the stresses is similar between the FEA model and the analytical solution. The next figure shows the deflections of the C5 lens. In the analytical solution of thickness of 55.1mm was used which is the thickness of the lens at its centerline. The calculated axial deflection of 0.0014" matches very well with the FEA deflection of 0.00105".

Figures 3 and 4 below show the temperature distribution on the T3 and T4 Surfaces respectively on the C5 lens. There is good agreement between the finite difference model described in Section 1.0 above and the FEA model.

It should be noted that the temperature gradients within C5 are heavily dependent upon a many different factors such as:
-radiation between C4 and C5
-the N2 flow rate
-the N2 input temperature
-the thermal conduction at the outer radius. A very simple model of the thermal conduction at the outer radius of C5 was used in this analysis in which there was only contact between C5 and 2 o-rings that were each 3/16" in diameter.

The stresses and deflections however are dominated by the pressure and not the temperature distributions within the lens. The calculations and FEA model it is felt accurately model the stresses and deflections of the lens. If more accurate temperature distributions are needed then a more detailed FEA model will have to be created that more accurately models the actual contact at the outer radius and models the entire assembly in a manner done by the finite difference model. Also, further thought will have to go into what a practical N2 flow rate is (3.0cfm was used in this calculation) and whether or not the N2 inlet temperature will be heated. In this analysis the temperature of the air, surrounding steel structure, and N2 inlet temperature was all set to 25C. Even slight changes in the N2 inlet temperature and flow rate could result in a very low N2 outlet temperature that could increase the risk of condensation on the lenses.

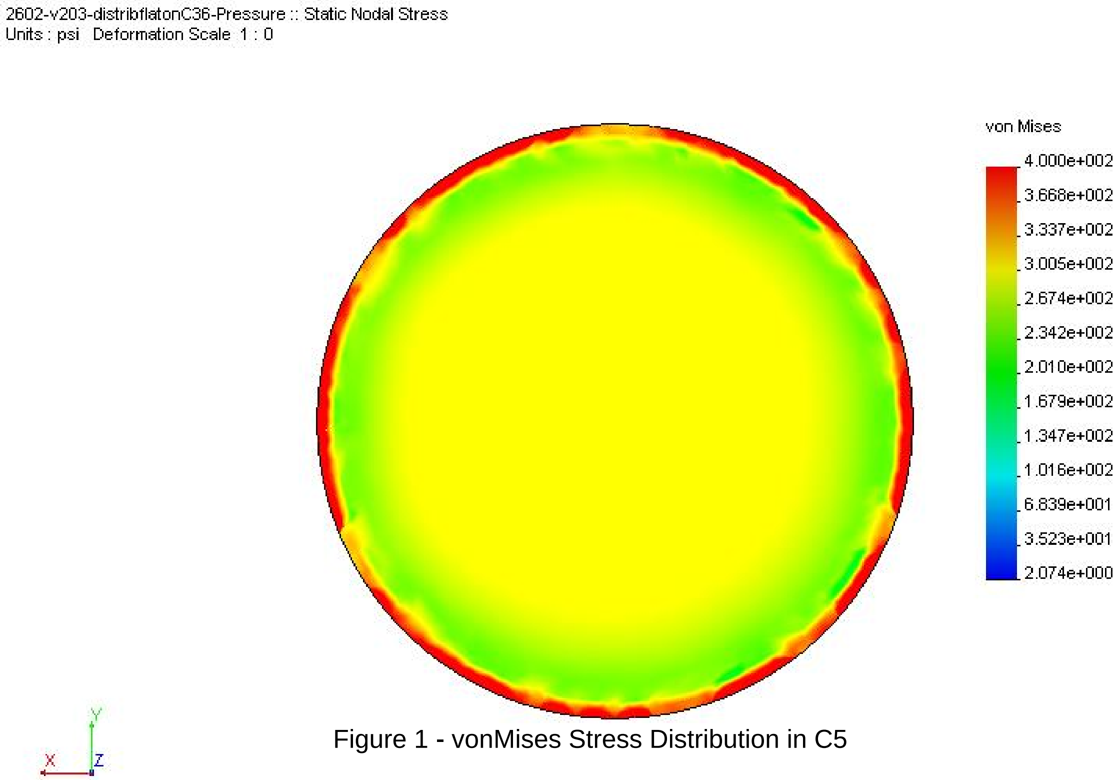


Figure 1 - vonMises Stress Distribution in C5

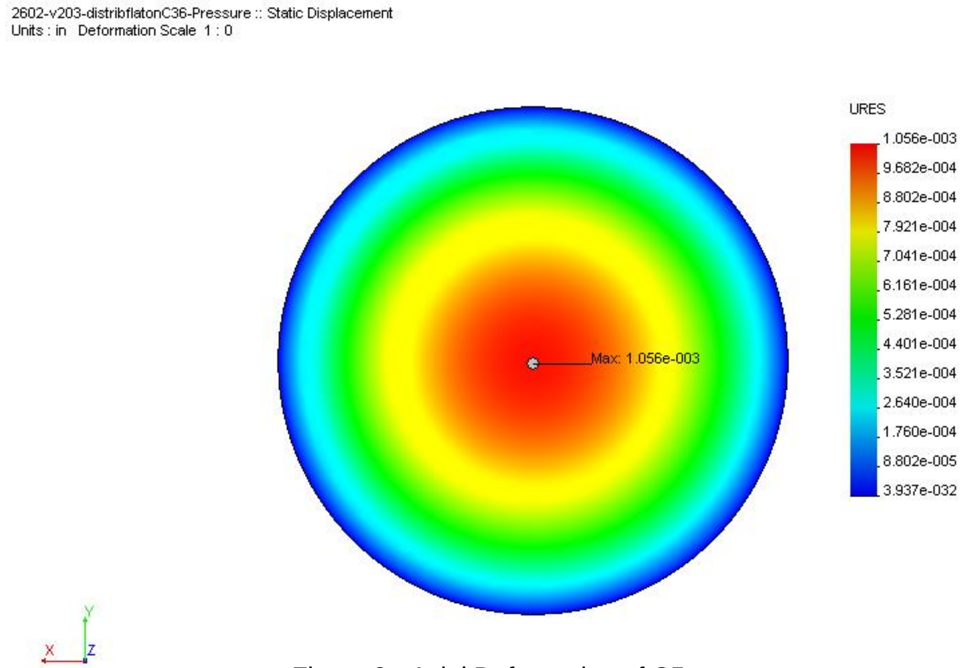


Figure 2 - Axial Deformation of C5

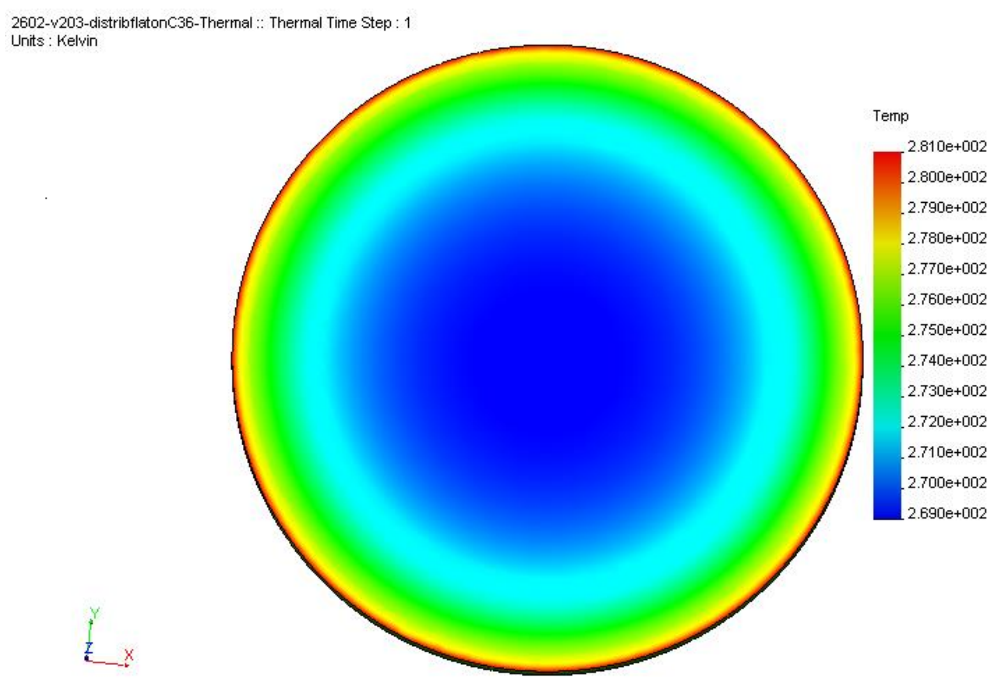


Figure 3 - Temperature Distribution on T3 Surface of C5

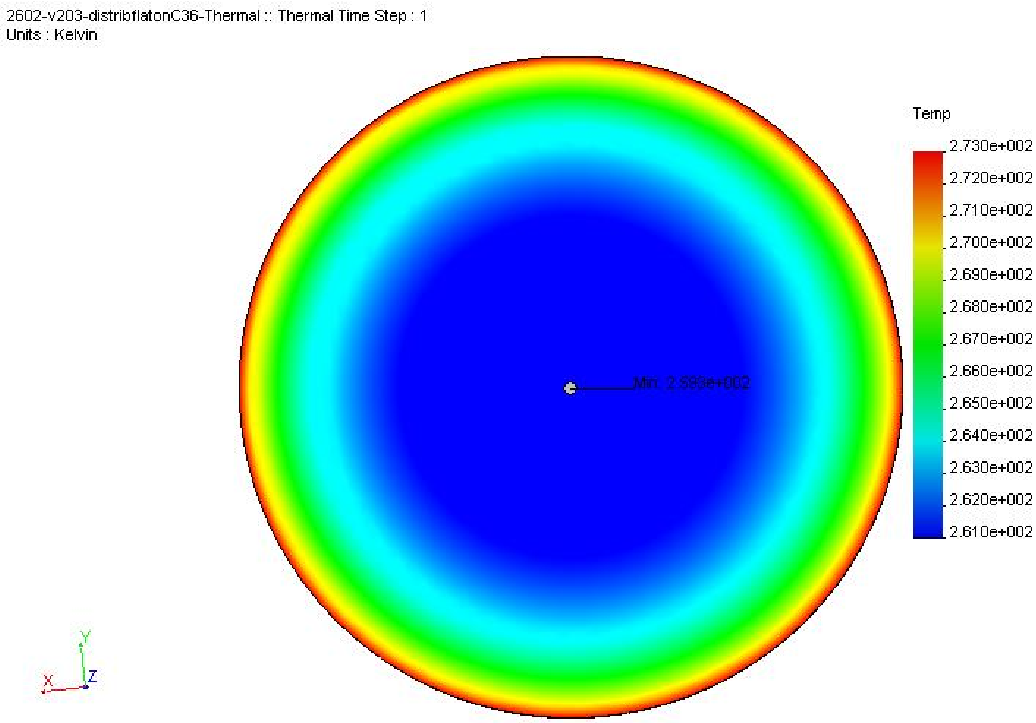


Figure 4 - Temperature Distribution on T4 Surface of C5 Lens

A comparisionn was made of the temperature distribution for various atmospheric temperatures. In all of these comparisions the atmospheric, support steel, and inlet N2 temperatures (Ta, Ts, Ti) were equal.

Temperatures when Ta=Ts=Ti=25C

$$\mathrm{T1a} = \begin{pmatrix} 287.715 \\ 287.890 \\ 288.420 \\ 288.848 \\ 289.409 \\ 290.135 \\ 291.060 \\ 292.211 \end{pmatrix} \mathrm{K} \qquad \mathrm{T2a} = \begin{pmatrix} 286.026 \\ 286.205 \\ 286.749 \\ 287.193 \\ 287.781 \\ 288.554 \\ 289.565 \\ 290.886 \end{pmatrix} \mathrm{K} \qquad \mathrm{T3a} = \begin{pmatrix} 271.756 \\ 272.050 \\ 272.973 \\ 273.763 \\ 274.847 \\ 276.329 \\ 278.343 \\ 281.034 \end{pmatrix} \mathrm{K} \qquad \mathrm{T4a} = \begin{pmatrix} 262.465 \\ 262.787 \\ 263.806 \\ 264.686 \\ 265.909 \\ 267.619 \\ 270.037 \\ 273.518 \end{pmatrix} \mathrm{K} \qquad \mathrm{Tna} = 290.817\mathrm{K}$$

Temperatures when Ta=Ts=Ti=10C

$$T1a = \begin{pmatrix} 272.551 \\ 272.780 \\ 273.437 \\ 273.932 \\ 274.550 \\ 275.307 \\ 276.214 \\ 277.275 \end{pmatrix} K \quad T2a = \begin{pmatrix} 271.036 \\ 271.277 \\ 271.974 \\ 272.503 \\ 273.170 \\ 273.998 \\ 275.009 \\ 276.224 \end{pmatrix} K \quad T3a = \begin{pmatrix} 259.525 \\ 259.810 \\ 260.682 \\ 261.402 \\ 262.369 \\ 263.660 \\ 265.373 \\ 267.611 \end{pmatrix} K \quad T4a = \begin{pmatrix} 252.011 \\ 252.315 \\ 253.254 \\ 254.039 \\ 255.107 \\ 256.566 \\ 258.582 \\ 261.417 \end{pmatrix} K \quad Tna = 277.390K$$

Temperatures when Ta=Ts=Ti= -5C

$$T1a = \begin{pmatrix} 261.719 \\ 261.819 \\ 262.116 \\ 262.352 \\ 262.661 \\ 263.057 \\ 263.560 \\ 264.184 \end{pmatrix} K \quad T2a = \begin{pmatrix} 260.518 \\ 260.624 \\ 260.940 \\ 261.194 \\ 261.528 \\ 261.962 \\ 262.524 \\ 263.254 \end{pmatrix} K \quad T3a = \begin{pmatrix} 248.037 \\ 248.265 \\ 248.965 \\ 249.545 \\ 250.324 \\ 251.362 \\ 252.736 \\ 254.522 \end{pmatrix} K \quad T4a = \begin{pmatrix} 241.976 \\ 242.225 \\ 242.990 \\ 243.630 \\ 244.496 \\ 245.672 \\ 247.281 \\ 249.517 \end{pmatrix} K \quad Tna = 263.352K$$

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